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SERIES A:—SCIENCE AND MATHEMATICS.

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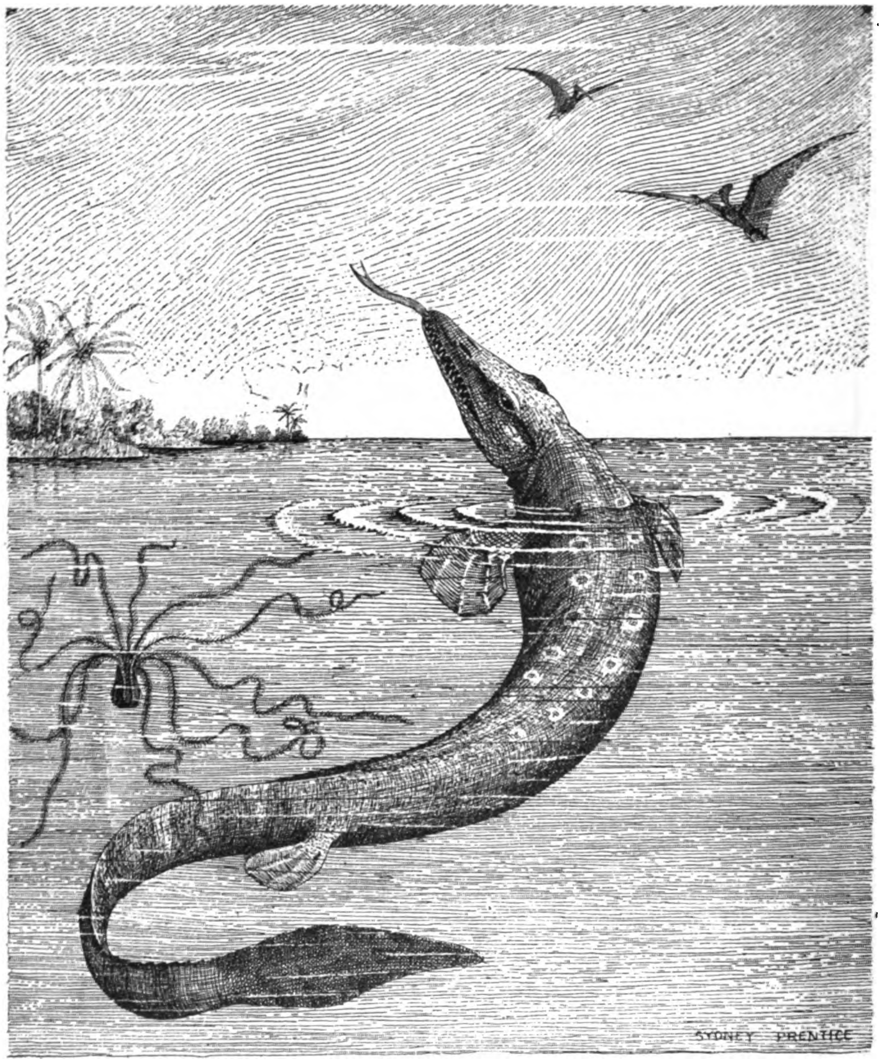


Uintacrinus. **Clidastes.** **Ornithostoma.**

RESTORATION OF KANSAS CRETACEOUS ANIMALS.

Drawn by Sydney Prentice, under the direction of S. W. Williston.

FRONTISPIECE, KAN. UNIV. QUAR. VOL. VII SERIES A.

KANSAS UNIVERSITY QUARTERLY.

VOL. VII. JANUARY, 1898. NO. 1.

Experiments in Judging the Distance of Sound.*

BY C. E. SHUTT.

The object of the experiments described in this paper was to determine how accurately the distance at which a sound is produced can be judged, and how much variation in judgment, if any, is caused by changing the position of the body relative to the direction from which the sound proceeds.

A chalk line fifty feet in length was drawn on the floor of a large room. This was marked off into distances of one foot. The person experimented upon sat blindfolded at one end of this line. In order to cover up the noise made by the movements of the person conducting the experiment, the subject kept striking the arm of the chair with a small block of wood, pausing at intervals to hear the sound whose distance he was to estimate.

The person performing the experiment moved back and forth along the line using a telegraph snapper and an A pitch pipe to make the sounds. No regular order was observed in selecting the distances for producing the sounds, nor was the subject aware of the distances at which his judgments were recorded. He was told, however, that no sound would be produced beyond fifty feet.

All subjects were tested in four positions. These were: (1) with the right side towards the sound; (2) with the left side towards the sound; (3) with the subject facing the sound; and (4) with the back towards the sound. Judgments were recorded every five feet. The telegraph snapper and pitch pipe were used with no

*Read before the Kansas Academy of Science at its annual session Oct. 29, 1897.

regular order of alternation until the record of judgments was complete in each case for both. Eighty readings were taken for each person, forty with each instrument used.

Twenty persons were tested. They were all students in the University of Kansas where the experiments were performed. Members of both sexes were included in the number.

In Table I are given the number of correct estimates, over-estimates and under-estimates made at each point in the line where judgments were recorded. The sum of each column is also expressed at the foot in terms of the percentage of all the readings taken in the position in which it stands.

TABLE I,

(a) Telegraph Snapper.

	Dis.	R.			L.			F.			B.			Av.		
	Ft.	C.	+	—	C.	+	—	C.	+	—	C.	+	—	C.	+	—
	5	8	12		6	12	2	1	17	2	4	14	2	4.0	13.7	1.5
	10	6	11	3	7	10	3	6	11	3	4	15	1	5.7	11.7	2.5
	15	8	8	4	3	12	5	2	15	3	6	12	2	4.7	11.7	3.5
	20	7	10	3	3	12	5	3	12	5	5	12	3	4.5	11.5	4.0
	25	4	8	8	7	7	6	3	12	5	5	12	3	4.7	9.7	5.4
	30	5	7	8	7	9	4	3	12	5	4	9	7	4.7	9.2	6.0
	35	3	9	8	3	8	9	6	8	6	2	10	8	3.5	8.7	10.2
	40	3	5	12	3	8	9	4	4	12	2	6	12	3.0	5.7	11.2
	45	4	4	12	4	6	10	5	4	11	4	2	14	4 2	4.0	11.7
	50	5		15	2		18	3		17	2		18	3 0		17.0
Total	...	53	74	73	45	84	81	36	95	69	38	92	70	42	85	73
Total in per ct.	..	26.5	37.0	36 5	22.5	42.0	40 5	18 0	47.5	34.5	19.	46.0	35.0	21.0	42 5	36.5

(b) Pitch Pipe.

Dis.	R.			L.			F.			B.			Av.			Gen. Av.		
Ft.	C.	+	—	C.	+	—	C.	+	—	C.	+	—	C.	+	—	C.	+	—
5	3	17	....	2	16	2	3	17	....	3	16	1	2.7	16.5	.7	3 3	15.1	1.1
10	1	18	1	2	17	1	1	18	1	2	17	1	1.5	17.5	1.0	3.6	14.6	1.7
15	3	15	2	3	14	3	....	19	1	....	19	1	1 5	14.2	1.7	3.1	12.9	2.6
20	2	17	1	....	17	3	1	18	1	4	15	1	1.7	16.7	1 5	3.1	14.1	2.7
25	3	14	3	3	14	3	1	18	1	2	15	3	2.5	15.2	2.5	3.6	12.4	3.9
30	5	11	4	4	11	5	3	14	3	1	15	4	3.2	12.7	4.0	3.9	10.9	5.0
35	4	10	6	1	9	10	2	13	5	1	11	8	2.0	13.2	7.2	2.7	10.9	8.7
40	7	4	9	4	10	6	5	8	7	3	6	11	4 7	7.0	8 2	3.8	6.3	9.7
45	3	7	10	3	5	12	5	2	13	3	5	12	3.5	4.7	11.7	3.8	4.3	11.7
50	6		14	2	...	18	7	.	13	3	....	17	4.5	......	15.5	3 7	...	16.2
To.	37	113	50	24	113	63	28	127	45	22	119	59	27 8	117.7	54 0	34 6	101.5	63.3
PC.	18.5	56 5	25.0	12.0	56.5	31.5	14.0	63.5	22.5	11.0	59.5	29.5	13.9	58.8	27.0	17.3	50.7	31.6

The two parts of this table agree in their results. The distance was correctly judged the greatest number of times with the right side towards the sound, and under-estimated the oftenest with the left side in that position. The distance was over-estimated oftenest when the subject was sitting with his face towards the sound. The number of over-estimates exceeds that of both the correct

judgments and the under-estimates. In part (b) of the table it exceeds the sum of the other two, as it does also in the general average.

Table II shows the maximum and minimum errors made at each distance in the four positions.

TABLE II.

(a) Telegraph Snapper.

	Dis.	R.		L.		F.		B.		Av.	
	Ft.	+	—	+	—	+	—	+	—	+	—
	5	15	......	11	1	10	1	13	1	12.2	.7
	10	15	3	10	3	12	4	10	2	11.7	3.0
	15	10	5	15	7	25	3	20	5	17.5	5.0
	20	15	5	15	10	15	10	15	5	15 0	7.5
	25	15	7	11	10	15	13	15	9	14.0	9.7
	30	15	12	25	10	20	16	13	10	15.7	12.0
	35	11	16	20	15	15	21	10	15	14 0	16.7
	40	8	20	18	17	10	23	10	22	11.5	20.5
	45	5	25	5	14	5	25	5	24	5.0	22.9
	50	......	30	......	26	......	29	...	20	.....	26.2
Total.		109	123	120	113	120	145	111	113	116 6	124 6

(b) Pitch Pipe.

	Dis.	R.		L.		F.		B.		Av.		Gen. Av.	
	Ft.	+	—	+	—	+	—	+	—	+	—	+	—
	5	23	...	40	2	35	..	45	..	35.7	.5	23.9	.6
	10	30	5	40	2	40	3	40	3	37.5	3 2	24 6	3.1
	15	30	10	35	7	35	7	35	5	33 7	7 2	25.6	6.1
	20	30	8	30	24	30	8	30	8	30.0	12.0	22.5	9.7
	25	25	5	20	13	20	10	25	13	22.5	10.2	18 2	9.9
	30	18	15	20	15	15	10	20	10	15.7	12.5	15.7	12.2
	35	15	20	15	10	10	15	15	15	13.7	15.0	13.8	15.8
	40	10	10	10	15	10	30	10	15	10.0	17.5	10.7	19.0
	45	10	30	5	20	5	15	5	25	6.2	22.5	5.6	22.2
	50	..	13	..	25	..	35	..	20	....	23 2	...	24 6
Total.......... ...		191	116	215	133	200	133	225	114	205 0	123.5	160.6	123.2

In (a) the sum of the maximum errors is the same with the left side and the face towards the sound, and is greater in those positions than in the other two. The sum of the minimum errors is the greatest with the face towards the sound. In part (b) of the table the sum of the maximum errors is the greatest with the back towards the sound, and that of the minimum errors is the same and greatest when the left side and face were turned towards it. The sum of the averages of the maximum errors is greater in (b) than in (a). The sum of maximum errors exceeds that of the minimum errors in the general average.

The next table contains the sum of the judgments of each sub-

ject in the four positions. In the first column of figures is given the sum of the distances at which records were made.

TABLE III.

(a) Telegraph Snapper.

	Dis.	R.	L.	F.	B.	Av.
A	275	188	178	166	153	171.2
B	2.5	223	191	288	233	233 7
C	275	337	295	348	345	331 2
D	275	296	234	252	278	265 0
E.	275	302	224	308	2 5	302.2
F	275	360	330	281	328	324.7
G	275	253	273	387	242	288.7
H.. ..	275	216	287	280	266	262.2
I	275	295	295	285	305	295.0
J......	275	272	311	318	324	306.2
K	275	283	261	320	280	286.0
L......	275	266	320	286	280	285 5
M.	275	270	302	300	352	306 0
N....	275	265	291	285	290	282 7
O	275	295	258	294	281	282.0
P.... ..	275	349	295	209	309	290.5
Q......	275	285	2 0	283	255	268.2
R......	275	147	331	260	294	283.0
S ...	275	256	202	225	249	233.0
T......	275	292	203	219	257	242.3

(b) Pitch Pipe.

	Dis.	R.	L.	F.	B.	Av.	Gen. Av.
A	275	229	200	289	244	240 5	205.8
B.. ..	275	311	330	342	325	329.2	281.4
C......	275	435	430	430	450	436.2	383.7
D.....	2.5	303	271	222	225	255 2	260.6
E.. ...	275	309	319	320	2 3	310.2	306 2
F	275	343	275	265	355	309.5	317.1
G ...	275	292	308	265	286	287.7	288.2
H.... .	275	270	392	335	263	290.0	276 1
I	275	340	340	36	290	332.5	313.7
J.	275	330	3 7	347	356	352.5	329 3
K . ..	275	230	370	327	286	303 2	294.6
L.. ...	275	398	490	445	519	463.0	374 2
M	275	281	295	344	334	313.2	319.6
N.. ...	275	331	365	328	311	333.7	308.2
O.......	276	215	190	164	220	197.2	239.1
P ..	275	378	240	358	336	328.9	309.2
Q	275	295	315	358	295	315.7	291.9
R......	2.5	261	332	345	311	312 2	297.6
S	275	305	285	287	342	3 7.2	270.1
T.......	275	327	336	300	250	303.2	272.7

From this table we deduce the following, which shows the number of subjects who gave respectively correct estimates, over-estimates and under-estimates:

	R.			L.			F.			B.			Total.		
	C.	+	—	C.	+	—	C.	+	—	C.	+	—	C.	+	—
(a) Tel. Snapper...		10	10		11	9	—	14	6	1	12	7	1	47	32
(b) Pitch Pipe......		15	5	1	15	4		16	4		15	5	1	61	18

Two subjects—C and J—made a complete record of over-estimates. The two correct averages were made by different persons. Subject A made a record of under-estimates in (a), but made an over-estimate in (b). Subject F made no under-estimates, but made a correct average in (b). The number of persons making over-estimates was the greatest when sitting with the face toward the sound. The number making under-estimates was the greatest when the right side was toward the sound. The latter position also favored the equal division between those making under-estimates and over-estimates. The number who over-estimated the distance was greater in every position in (b) than in the corresponding positions in (a). In general the number who made over-estimates was much greater than that of those who made under-estimates.

The averages of the estimated distances are given in the following table:

TABLE IV.

(a) Telegraph Snapper. (b) Pitch Pipe.

	Dis.	R.	L.	F.	B.	Av.	Dis.	R.	L.	F.	B.	Av.	Gen. Av.
	5	7.7	8.8	8.3	5.4	7.4	5	11.8	14.1	15.3	14.6	13.9	10.6
	10	13.3	12.2	12.5	1[illegible].6	13.4	10	20.3	22.6	22.7	21.0	21.6	17.5
	15	16.4	17.9	18.8	20.0	18.2	15	36.5	26.3	29.3	27.8	27.4	22.8
	20	22.8	21.8	22.0	23.9	22.6	20	30.5	29.9	30.7	29.7	30.2	26.4
	25	21.6	25.8	26.8	29.1	25.8	25	32.0	30.4	31.9	31.0	31.4	28.6
	30	27.5	30.1	32.0	31.1	30.1	30	33.0	34.5	33.9	34.6	34.0	32.0
	35	35.9	34.8	34.7	35.6	3[illegible].2	35	37.7	36.2	37.3	37.6	37.2	36.2
	40	38.7	37.7	36.0	37.3	37.4	40	35.8	40.4	37.1	39.4	38.1	37.7
	45	41.2	40.4	40.3	40.2	40.5	45	41.3	39.7	41.6	41.0	40.9	40.7
	50	42.0	42.1	41.7	42.3	42.0	50	42.8	42.0	42.6	40.0	41.8	41.9
Total....	275	271.1	271.1	274.1	279.5	272.6	275	311.7	316.1	322.4	316.7	316.5	294.4

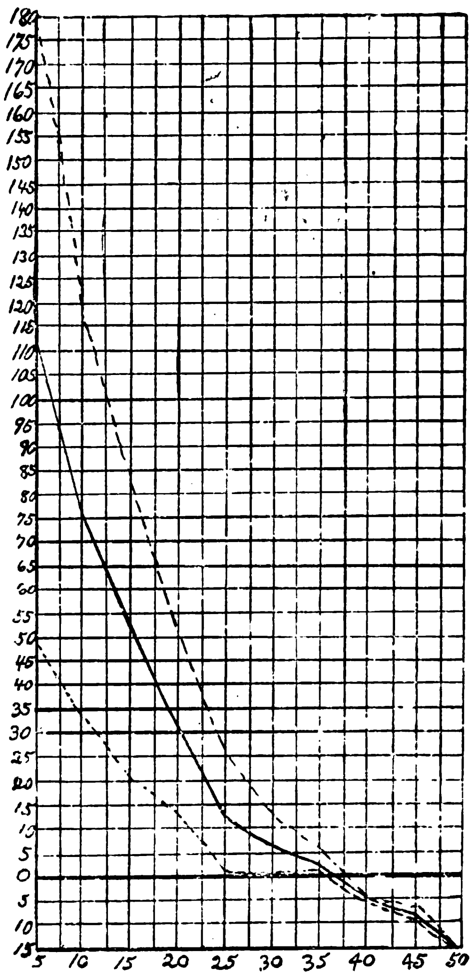
The sums of the averages in (a) are all fairly accurate, while those in (b) are much too large. The most favorable position in (a) for judging the distance was with the face toward the sound. In (b) the results obtained with the right side in that position are the nearest correct. The most unfavorable position in (a) was with the back toward the sound, and in (b) with the face toward it.

Table V gives the average errors stated in percentages of the distances at which the sounds were produced.

TABLES V AND VI.

(a) Telegraph Snapper. (b) Pitch Pipe.

Dis.	(a) R.		L.		F.		B.		Av.		(b) R.		L.		F.		B.		Av.		Gen. Av.	
Ft.	+	—	+	—	+	—	+	—	+	—	+	—	+	—	+	—	+	—	+	—	+	—
5	54.0		66.0		66.0		8.0		48.5		136.0		182.0		20.6		188 0		178.0		113.2	
10	33.0		22.0		35.0		46.0		34.0		103.0		126 0		127.0		110.0		116 5		75.0	
15	9.3		19.3		25.3		33.3		21.8		.76.3		75.3		95.3		85.3		83 3		52.5	
20	14.0		9.0		10.0		19.5		13.1		52 5		49.9		53.5		48.5		51.1		32.1	
25		15.6	3.2		6.2		12.4		1.5		28.0		21.6		27.6		24.0		25.3		13.4	
30		8.3	.3		6.6		3.3		.4		10.0		15 0		13 0		15.3		13.5		6.9	
35	2.6			6		.8	1.7		.7		7.7		3.4		6.3		7.4		6.2		3.4	
40		3.2		5.7		10.0		6.7		6 4		10.5	1.0			7.2		1.5		4.5		5.4
45		8.4		10.2		10.4		10.7		9.9		8.2		1.1		7.5		8.8		6.4		8.1
50		16.0		15.8		16.6		15.4		15.9		14.4		16.0		14.8		20.0		16.3		16.1



There is a tendency in both (a) and (b) toward overestimation within thirty-five feet. Beyond that distance the opposite tendency prevails.

These facts are shown to better advantage in the direction of the following curves. The dotted line represents the average in (a), the broken line that in (b), and the unbroken line the general average. The figures on the left side above the line marked. zero, represent percentages of overestimation, and those below it, per-

centages of underestimation. The figures at the bottom represent distances, at which judgments were recorded.

1. The distance of a sharp noise can be more accurately estimated than that of a smooth tune.

2. Judgment is the most accurate with the right side toward the sound and most inaccurate with the face toward it.

3. The tendency is to overestimate the distance of both a sharp noise and a smooth tone within thirty-five feet. Beyond that distance there is a tendency toward under-estimating.

4. Within thirty-five feet the tendency to overestimate the distance of a smooth tone is greater than it is to overestimate that of a sharp noise within the same distance.

Experiments in Judging the Direction of Sound.*

BY L. D. IKENBERRY AND C. E. SHUTT.

The experiments herein described were made for the purpose of determining the accuracy with which the direction of a sound can be determined.

A circle ten feet in diameter was drawn on the floor of a large room. From one end of a diameter marked zero, the right and left semi-circumferences of the circle were marked off into fifty equal parts. The subject was seated in the center of the circle facing the zero point. Besides being blindfolded, his head was bound to a support on the back of the chair in order to keep him from turning it during the experiments. Sounds were produced directly over certain points in the circle on a level with the subject's head. Immediately after the production of each sound the subject named the point in the circle over which he judged it to be located. No regular order was observed in selecting the points for producing the sounds.

The instruments used in making the sounds were a telegraph snapper and a common harmonicon. The telegraph snapper was used to make a sharp, piercing noise and the harmonicon to produce a smooth tone. Two sets of readings were taken with each instrument. In the first set both ears were open, but in the second his left ear was effectually closed.

Efficient means were employed whereby the subjects were made wholly dependent upon the sound itself in locating it. Ten persons were tested. Both sexes were included in the number.

In table I are given the judgments. The figures at the head of the columns indicate the points in their respective semi-circles over which the sounds were made. The other figures show where the subjects thought they were located. The letters on the first column represent the subjects. At the foot of each column are

*Read before the Kansas Academy of Science, at its annual session, Oct. 20, 1897.

given the average of the judgments, and the average error. Figures marked with a star were located in the wrong semi-circle.

TABLE I.

(a) Harmonicon.

BOTH EARS OPEN.

	Right.					Left.				
	8	15	25	35	42	8	15	25	35	42
A	15	24	23	27	23	12	17	25	36	39
B	18	22	30	40	48	15	10	30	35	45
C	16	25	25	42	30	10	25	30	33	40
D	12	20	32	38	35	10	25	30	20	45
E	18	21	24	28	35	8	20	25	37	40
F	20	19	28	27	35	11	25	25	37	44
G	12.5	20	13	40	45	5	25	25	33	45
H	12 5	23	25	37.5	30	12.5	15	30	32 5	37.5
I	15	18	20	20	35	15	15	25	30	40
J	17	21	20	28	34	5	13	12	40	50
Average	15.6	21.3	24	32 75	34.2	9 4	19	25.7	33.35	42.55
Too small	.....	.....	1	2.2	7 8	.....	.....	.....	1.65	.55
Too large	7.6	6 3	.....	.....	.....	1.4	4	.7	.....	

LEFT EAR CLOSED.

	Right.					Left.				
	8	15	25	35	42	8	15	25	35	42
A	25	19	19	25	19	14	33	30	36	46
B	17	15	23	20	40	10	12	30	40	50
C	25	45	32	30	26	20	17	14	34	42
D	20	15	20	37	37	*40	15	45	25	*20
E	25	20	4	2	* 2	3	35	35	40	*30
F	45	19	30	32	32	32	40	50	35	37
G	12.5	12.5	50	25	40	45	35	*45	50	45
H	35	12 5	25	37.5	25	10	30	17.5	37.5	25
I	25	20	23	35	25	5	25	25	25	0
J	20	25	25	25	18	3	35	37 5	0	0
Average	24.9	20.3	25.1	26.8	36 7	22.2	28.7	35.4	32.3	39.5
Too small	.....	.....	.....	8.2	5.3	.....	.....	.....	2.7	2.5
Too large	6.9	5.3	.1	.....	.....	14 2	13.7	10 4		

(b) Telegraph Snapper.

BOTH EARS OPEN.

	Right.					Left.				
	8	15	25	35	42	8	15	25	35	42
A	18	16	22	22	30	13	23	26	30	42
B	18	22	30	30	45	15	28	27	35	40
C	12	36	24	30	46	6	17	18	30	43
D	10	20	30	37	40	35	20	35	40	40
E	13	20	19	30	38	8	18	17	33	35
F	15	20	26	40	30	18	20	20	38	40
G	12 5	20	18	30	45	13	20	20	35	40
H	12 5	20	25	37.5	40	12 5	25	25	32 5	48
I	12	15	25	35	40	15	30	30	30	40
J	9	15	23	12	10	5	20	37	48	41
Average	13.2	20.4	24.2	30.3	36.4	14.1	22.1	26 5	35 2	40.9
Too small	.....	.....	.8	4 7	5 6	.	.....	.....	.....	1.1
Too large	5.2	5.4	.....	.....	.	6.1	7.1	1 5	.2	

LEFT EAR CLOSED.										
	RIGHT.					LEFT.				
	8	15	25	35	42	8	15	25	35	42
A	13	13	22	24	44	15	26	12	13	16
B	30	*30	27	20	30	50	10	30	35	45
C	15	10	20	20	42	2	10	17	8	43
D	10	25	35	38	40	50	15	30	30	48
E	21	24	18	33	45	3	32	18	40	13
F	25	25	25	23	25	0	8	20	22	20
G	48	20	20	12.5	22	45	33	38	35	25
H	12.5	37.5	23	25	12.5	0	12.5	37.5	37.5	50
I	35	30	20	25	35	20	25	30	35	35
J	10	11	21	24	21	2	5	25	37	0
Average	21.9	21.4	23.3	24.9	31.4	18.7	17.7	29.8	28.3	28.5
Too small	.....	.....	1.7	11.1	10.5	.....	.....	.....	6.7	13.6
Too large	13.9	6.4	.....	.....	.....	10.7	2.7	4.8		

The averages at points 8 and 15 on the right side were all too large, while the same is true of the points 8, 15 and 25 on the left side.

The following table shows the number of correct judgments, as well as the erroneous ones made by each subject. c, b. and f at the head of the columns stand respectively for correct judgments, those too far back, and those too far in front.

TABLE II.

	HARMONICON.												TELEGRAPH SNAPPER.												TOTAL.		
	Both ears open.						Left ear closed.						Both ears open.						Left ear closed.								
	R.			L.			R.			L.			R.			L.			R.			L.					
	c	b	f	c	b	f	c	b	f	c	b	f	c	b	f	c	b	f	c	b	f	c	b	f	c	b	f
A	..	2	3	1	3	1	..	2	3	..	5	..	..	2	3	1	3	1	1	2	2	..	2	3	3	21	16
B	..	5	..	1	3	1	1	1	3	..	4	1	..	4	1	1	3	1	..	3	2	1	3	1	4	26	10
C	1	3	1	.	3	2	..	3	2	1	2	2	..	3	2	..	2	3	1	1	3	..	1	4	3	18	19
D	..	4	1	..	4	1	1	2	2	1	2	2	..	4	1	..	4	1	..	4	1	1	3	1	3	27	10
E	..	2	3	2	2	1	..	2	3	..	3	2	..	2	3	1	1	3	..	3	2	..	2	3	3	17	20
F	..	3	2	1	4	..	..	3	2	1	3	1	..	4	1	..	3	2	1	2	2	..	..	5	3	22	15
G	..	4	1	1	2	2	..	2	3	..	5	..	..	3	2	1	2	2	..	2	3	1	3	1	3	23	14
H	1	3	1	1	2	2	1	2	2	..	3	2	1	3	1	1	3	1	..	2	3	..	3	2	5	21	14
I	..	2	3	2	1	2	1	2	2	1	1	3	3	1	1	.	3	2	..	2	3	1	3	1	3	15	17
J	..	2	3	..	2	3	1	2	2	..	2	3	1	2	3	..	3	2	..	1	4	1	1	3	3	15	23

Six subjects—A, B, D, F, G, and H, located the sounds too far back as a rule. Four—C, E, I, and J, had the opposite tendency.

Table III contains the number of correct judgments, those too far back, and those too far in front at each point where sounds were produced in each semi-circle.

TABLE III.

	HARMONICON.												TELEGRAPH SNAPPER.												TOTAL.		
	Both ears open.						Left ear closed.						Both ears open.						Left ear closed.								
	R.			L.			R.			L.			R.			L.			R.			L.					
Dis.	c	f	b	c	f	b	c	f	b	c	f	b	c	f	b	c	f	b	c	f	b	c	f	b	c	f	b
8	..	10	..	1	7	2	..	10	..	..	7	3	..	10	..	1	7	2	..	10	..	.	5	5	2	66	12
15	..	10	..	2	6	2	2	6	2	1	8	1	2	8	..	..	10	..	1	7	2	1	4	5	9	59	12
25	2	3	5	5	4	1	2	3	5	1	7	2	2	3	5	1	5	4	1	2	7	1	5	4	15	32	33
35	..	5	5	1	4	5	1	2	7	1	5	4	1	3	6	2	3	5	.	1	9	3	3	4	9	26	45
41	..	2	8	..	5	5	..	..	10	1	3	6	..	3	7	1	2	7	1	2	7	...	4	6	3	21	56
	2	30	18	9	26	15	5	21	24	4	30	16	5	27	18	5	27	18	3	22	25	5	21	24	38	204	158

The greatest number of correct judgments was made at the point 25, where the sound was at one side of the subject. The largest number of those too far back was made at point 8, and of those too far in front at 42. These points were respectively the greatest distances in front and behind the subject at which sounds were produced. The right side exceeds the left in the number of underestimates.

The totals in Table III may be managed as follows:

	Both ears open.						Left ear closed.						TOTAL.		
	R.			L.			R.			L.					
	c	f	b	c	f	b	c	f	b	c	f	b	c	f	b
Harmonicon..	2	30	18	9	26	15	5	21	24	4	30	16	20	107	73
Tel. Snapper..	5	27	18	5	27	18	3	22	25	5	21	24	18	97	85

The number of correct judgments and judgments locating the sounds too far back was larger when the harmonicon was used than when the sounds were made with the telegraph snapper. The number of judgments in which the sounds were located to far in front was greater with the use of the latter instrument.

Arranging the same figures differently we obtain the following result:

	HARMONICON.						TELEGRAPH SNAPPER.						TOTAL.		
	R.			L.			R.			L.					
	c	f	b	c	f	b	c	f	b	c	f	b	c	f	b
Both ears open...	2	30	18	9	26	15	5	27	18	5	27	18	21	110	69
Left ear closed ..	5	21	24	4	30	16	3	22	25	5	21	24	17	94	89

Correct judgments and judgments locating the sounds too far back were favored when the subjects used both ears. But the

number of judgments locating the sound too far in front was greater when one ear was closed than when both were open. In both cases, however, the tendency was to locate the sounds too far back.

By arranging the judgments recorded in front and back of point 25 we have this set of figures:

	HARMONICON.												TELEGRAPH SNAPPER.												TOTAL.		
	Both ears open.						Left ear closed.						Both ears open.						Left ear closed.								
	R.			L.			R.			L.			R.			L.			R.			L.					
	c	f	b	c	f	b	c	f	b	c	f	b	c	f	b	c	f	b	c	f	b	c	f	b	c	f	b
Front.	..	20	..	3	13	4	2	16	2	1	16	3	2	18	..	1	17	2	1	17	2	1	9	10	11	136	23
Rear..	..	7	13	1	9	10	1	2	17	2	8	10	1	6	13	3	5	12	1	3	16	3	7	10	12	47	101

The number of correct judgments was nearly the same in both positions. That of those locating the sound too far back was the larger when the experimenter was in front of the subject. Judgments locating it too far in front prevailed when the experimenter was back of him. The whole number of judgments locating the sound back of the point 25, is larger than that of those locating it in front of that point. This fact indicates a general tendency to locate sounds too far back.

The supposed location of the sounds made at points 0 and 50 varied so much that a separate arrangement of them is required. The table below gives a summary of the supposed locations. The columns marked C contain the correct judgments. Those marked with an interrogation point show how often the subjects were in doubt whether the sound was in front of or behind them. Columns with an X at the top indicate the number of times sounds located in front of the subject were supposed to be behind him and *vice versa*. R and L mean that errors were made respectively toward the right and left.

TABLE IV.

	HARMONICON.										TELEGRAPH SNAPPER.										TOTAL.				
	0					50					0					50									
	C	?	X	R	L	C	?	X	R	L	C	?	X	R	L	C	?	X	R	L	C	?	X	R	L
Both ears open,	5	3	..		2	..	..	3	4	3	4	4	..	1	1	5	..	..	2	3	14	7	3	7	9
Left ear closed.	2	3	..	3	2	2	1	2	4	1	1	4	..	3	2	3	..	2	2	3	8	8	4	12	8
	7	6	..	3	4	2	1	5	8	4	5	8	..	4	3	8	..	2	4	6	22	15	7	19	17

More errors were made at these points when the left ear was closed than when both were open. The same number of errors in the aggregate was made with the two instruments, and nearly the same at the two points 0 and 50. Upon the whole there was a slight inclination to err towards the right.

The following diagrams show the points at which each sound was located. The Roman numerals indicate the points where the sounds were produced. The figures at the end of the heavy lines

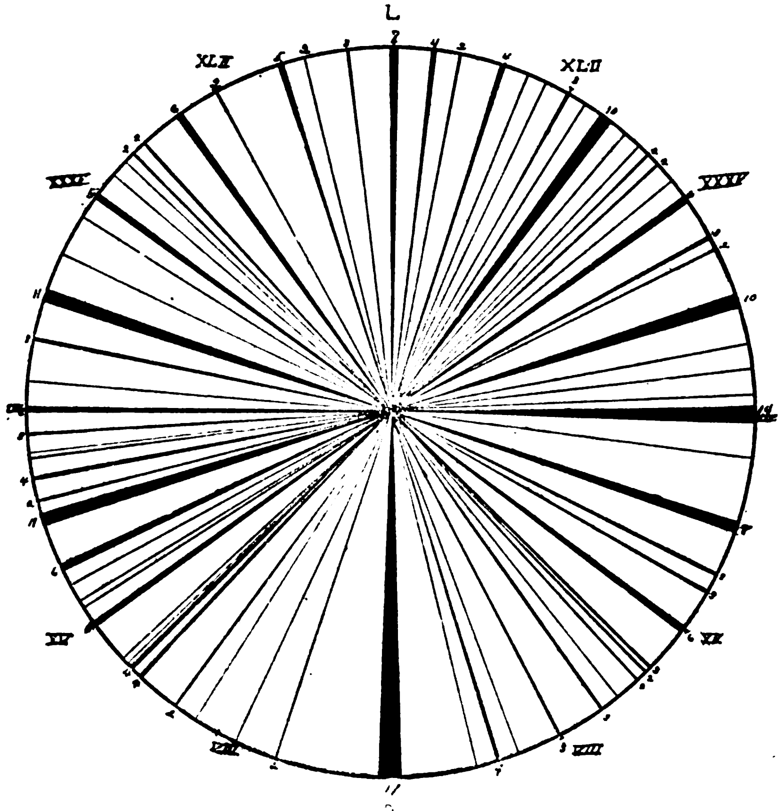


Fig. (a).

represent the number of sounds supposed by the subjects to be located at those points. (A) represents the supposed locations of the sounds when the subjects used both ears, and (b) when they used only the right ear. As there are two hundred and forty lines in each diagram and twelve points where sounds were made, twenty lines would terminate at each point, if all the sounds had been correctly located by the subjects. The tendency in the diagrams to vary from this regular arrangement shows the general trend of errors in locating the sounds.

The right semi-circle in (a) contains eighteen more lines than the corresponding semi-circle in (b). Between the other corresponding semi-circles there is very little difference in the number of lines. In the right front quadrant in both diagrams there is a point between forty-five and ninety degrees where a considerable number of lines congregate. The zero point in (a) gathers to it all the lines for a considerable distance on both sides of it. In the rest of the circle the lines are more or less scattered. The rear semi-circle in both diagrams has a larger number of congregated lines than the semi-circle in front. The lines in both diagrams also

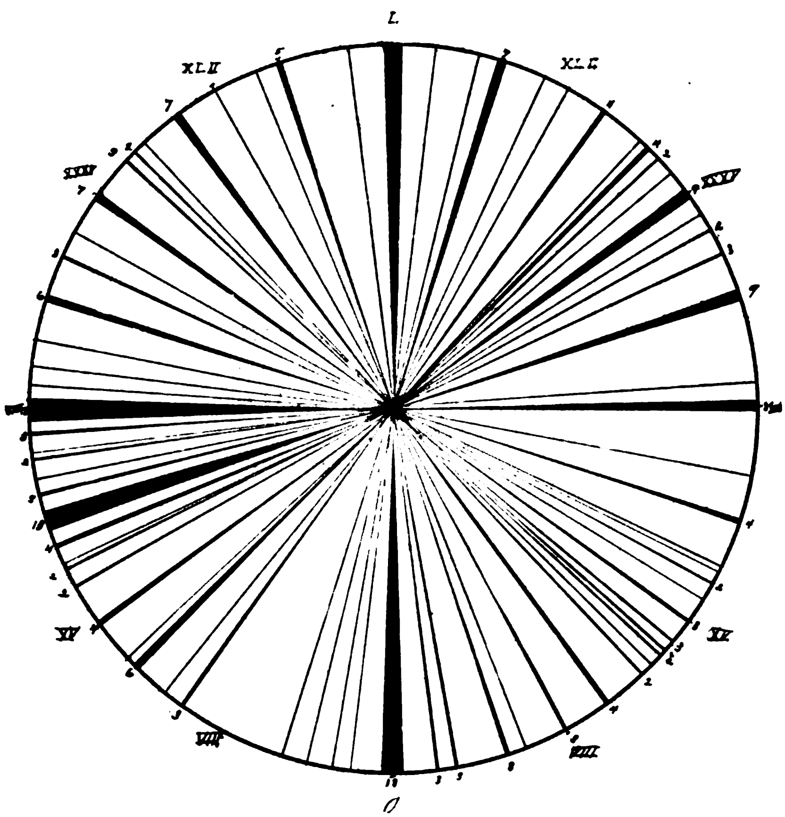


Fig. (b).

show an inclination to collect about the two points marked XXV. They have an evident preference to group together about certain points.

GENERAL CONCLUSIONS.

1. A sound can be the most accurately located when it is at one side of the person.

2. A sound produced back of a person is likely to be judged too far towards the front, and one in front too far back.

3. The direction of a smooth tone can be more accurately determined than that of a short, piercing one. The greatest error in both cases is that of locating the sounds too far back.

4. The direction of a sound can be determined with greater accuracy when both ears are used than when one is closed.

5. The position in which the direction of a sound can be most accurately determined is with the sound at the left with both ears open. That in which the greatest errors are likely to be made is with the sound at the left with the left ear closed.

6. There is an inclination upon the whole to locate sounds too far back.

New Corals from the Kansas Carboniferous.*

BY J. W. BEEDE.

Amplexus westii, n. sp.

Corallum simple, sub-cylindrical or attennuate conical, curved or geniculated, longitudinal striæ prominent, concentric lines of growth distinct, epitheca thin; septa extending one-half distance to center, eighteen to twenty-four or more in number; counterseptum somewhat longer than the others, which are about equal; indications of secondary septa visible, but not more than ½ mm. in length as seen in transverse section. Tabulæ well developed, 1½ to 3 mm. distant, reaching from wall to wall; on leaving the wall they are directed obliquely upward a short distance then slightly arching and undulating cross the center; occasionally branched at or near the bend. Length of specimen about 47 mm. diameter, in larger part, 9 mm.

Carboniferous, Upper Coal Measures, Kansas City. Collected by Judge E. P. West.

Cladochonus bennetti n. sp.

Corallum loosely fasciculate; corallites one to two or more diameters distant, erect coralites larger than basal branches, often five times as high as thick, upper portion budding and sending off branches as at base; epitheca strongly wrinkled, upper portion of wall of calyx thin, opening circular, deep, funnel-shaped by thickening of wall of corallite interiorly, in the lower portion of which is only a small capillary opening through the center. Average diameter of corallite 2 mm.; length, 6 to 18 mm.

This specimen resembles *Romingeria* (*Quenstedtia* Rom.) *umbellifera* Rom., but the absence of tabulæ removes it from that genus. It agrees with *Cladochonus* McCoy (*Pyrgia* E. and H.) save that the corallites are only funnel-shaped when young. The corallites are long and very stout, resembling *Syringopora* in outward appearance.

So far as I am aware this is the first time the genus has been reported from the United States. Carboniferous, Lower Coal Measures, Fort Scott. Presented by Rev. John Bennett.

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Aulopora? anna, n. sp.

Corallum prostrate, diffusely branched, branches interlacing, anastomosing at every contact, walls thin save at base of corallite, tubes very short, slightly sub-conical, immediate openings slightly flaring, circular to oval; no tabulæ distinguishable; septa occasionally represented by a faint ridge in best preserved corallites; diameter of calyx opening 2 mm., contiguous to one or two diameters distant. Corallites moderately low, larger at upper extremity than at base.

This species is profusely branched baso-laterally and anastomoses to such an extent as to often form solid mats of coral. It differs from *Aulopora* in having no tabulæ, in which respect it agrees with *Cladochonus*, but it is prostrate, and does not reproduce by lateral gemmation as does the latter, hence it is referred provisionally to *Aulopora*. Carboniferous, Upper Coal Measures Morehead, Kansas.

Aulopora prosseri, n. sp.

Corallum large, prostrate, bifurcating, calyces rising vertically or obliquely from 3 to 7 mm.; average diameter 2 mm. or less, average diameter of prostrate position slightly less. Calyces not campanulate, cylindrical, openings nearly circular; corallites wrinkled, weathered specimens show longitudinal striæ indicating rudimentary septa; distance of corallites, 1 to 3 diameters; in lower portion the cavity is nearly closed by internal thickening of wall. Tabulæ very remote or wanting, depressed when present.

Another specimen, apparently of this species, has a transverse weathered surface in which the corallites seem to be nearly closed by annular deposits within, there being merely a capillary opening in the center—a longititudinal section shows calyces 3½ to 5 mm. deep, thin walled to near base, where the walls rapidly thicken. One corallite in the section measures 15 mm. in length.

Carboniferous, Upper Coal Measures, Lyndon, Osage Co., Kansas.

EXPLANATION OF PLATE.[1]

1. *Cladochonus bennetti*[2] n. sp. Page 17. X $\frac{2}{1}$.
2. *Aulopora prosseri* n. sp. Page 18. Natural size.
3. *Aulopora anna* n sp. Page 18. Natural size.

"12." *Amplexus westii* n sp. Page 17. Natural size.

1 To accompany "New Corals from the Kansas Carboniferous" in previous number. Page 17.

2 Since writing the article I have been informed by Professor Charles S. Prosser that Professor H. S. Williams has noted this genus from the Devonian (Portage and Chemung) of New York. See Bull. U. S. G. S. No. 3, pp. 14 and 24.—J. W. B.


12

A Geological Map of Logan and Gove Counties.

BY GEO. I. ADAMS.

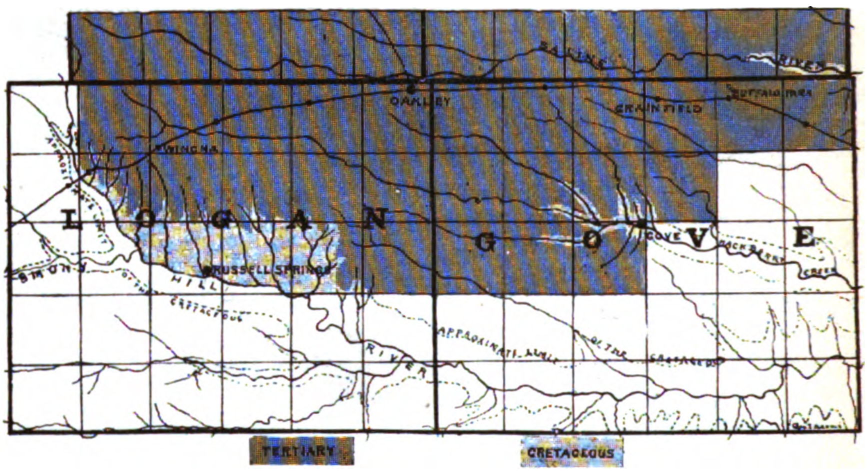
The accompanying map is published as a record of field work done in the summer 1896, while the writer was engaged in investigating the water supply of the area around Oakley, (see Report of Kansas State Board of Irrigation 1896-7, p. 113) and is here presented, with the hope, that it may assist in the further mapping of the geological formations. The shaded portions of the map are as accurate as the scale will permit, the field mapping having been referred to section lines or streams. Logan and Gove counties are here shown, also the southern row of townships of Sheridan and Thomas counties.

The level plain around Oakley, is a portion of the Tertiary formation of the western portion of the state. In this area it has a thickness of not over 200 feet. The formation consists of clays and sand, more or less cemented and mixed with some gravel. It has been eroded along the principal streams exposing the underlying formation which is the Cretaceous.

Northeast of Buffalo Park, in Sec. 22, T. 10, R. 27, is the western limit at which the Niobrara is exposed along the Saline. There it occupies the immediate valley of the stream. The area gradually widens to the east. In the vicinity of Gove the Niobrara is shown on the map by the light shading. The outcrops become more important to the east, as is indicated by the dotted lines. The Niobrara is easily recognized, being the "chalk" formation.

The area, shown in light shading north of the Smoky Hill river, at Russell Springs, is for the most part Fort Pierre. The Niobrara is seen in the valley as far west as the vicinity of Russell Springs, but occupies a limited belt. The line between the two fermations was not traced. The Fort Pierre shales erode quite easily thus

producing a bluff several miles back from the river. The formation may be recognized by the blue shales and the concretions or septaria which they contain. It is typically exposed at McAlister.



A Contribution to the Knowledge of the Ichthyic Fauna of the Kansas Cretaceous.*

BY ALBAN STEWART.

With Plates I and II.

Since the publication of Prof. Cope's Cretaceous Vertebrata in 1875, very little work has been done upon the Teleost fishes from the Kansas Cretaceous. During the time intervening since then four papers have appeared upon this Subclass, two in America by Cope†, ‡, and two in Europe by Felix‖, and Crook§, The last two have thrown much light upon the osteology of the forms treated, but only one new species was described, *Ichthyodectes polymicrodus* Crook, which Cope thought to be a synonym of his *I. arcuatus***, although this fact cannot be determined until a more complete description and a figure are made of the type of this species. The object of the present paper is to describe five new species from the Cretaceous of this state, with a few remarks upon the classification of the Saurodontidae, which I think are justifiable in the light of some new characters shown by the material in the Kansas University Museum.

Family Saurodontidæ Cope

Saurocephalidæ Zittel

Ichthyodectidæ Crook.

Cope characterizes this family as embracing carnivorous fishes, many of large size, and as being the predominant type of fishes during the Cretaceous period of North America. He gives the following synopsis††:

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†Bull U. S. Geol. Surv. Terr.., Vol. III, No. 4, p. 821.

‡Proc. Am. Phil. Soc., Vol XVII, p. 176-181. It might be well to mention Cope's description of *Syllæmus latifrons* in this connection: Rep. U. S. Geo. Surv., West One Hundreth Mer., Part II, Vol. IV, p. 27.

‖Zeit. d. Deut. Geol. Ges. Band XLII. p. 278-302.

§Paleontographica 1892, p. 107-124.

**Am. Nat., Vol. XXVI, p. 941.

††Cret Vert., p. 198.

I. Jaws without foramina on the inner face below the alveolar margin:

a. Teeth cylindric:
Teeth of unequal lengths; some of them greatly developed . . . . *Portheus.*
Teeth of equal length . . . . *Ichthyodectes*

aa. Teeth compressed, knife like:
Teeth of unequal length; some of the anterior greatly developed . . . . *Erisicthe.**
Teeth equal . . . . *Daptinus.*

II. Dentary bones pierced by foramina below the alveolar border:
Teeth with sub-cylindric crowns . . . . *Saurodon.*
Teeth with short, compressed crowns . . . . . . *Saurocephalus.*

Prof. Cope also says†: "There are some other forms to be referred to this family, whose characters are not yet fully determined. Thus *Hypsodon* Agass., from the European Chalk, is related to the two genera first above named, but, as left by its author in the 'Poissons Fossiles,' includes apparently two generic forms. The first figured and described has the mandibular teeth of equal length. In the second, they are unequal, as in *Portheus,* to which genus this specimen ought, perhaps, to be referred. Retaining the name *Hypsodon* for the genus with equal mandibular teeth, its relations to *Ichthyodectes* remain to be determined by further study of *H. lewesiensis.* The view of the superior walls of the cranium given by Professor Agassiz presents characters quite distinct from what I have observed in *Portheus.*"

In the light of the above and having compared Agassiz's type specimens with remains of *Portheus* and *Ichthyodectes,* Mr. E. T. Newton says‡: "I am convinced of the necessity of dividing *Hypsodon lewesiensis* as suggested by Prof. Cope; and it is proposed to retain this name for the specimen first described by Agassiz, and upon which the species and genus are really founded (Poiss. Foss., Vol. V, pl 25a, figs, 1, 2, and 4), and to remove Dr. Mantell's specimens and certain others (Poiss. Foss., pl. 25a, fig. 3, and 25b, figs. 1a, 1b, 2, and 3), to the genus *Portheus* of Cope."

*The name *Erisicthe* Cope. 1872, has been shown to be a synonym of *Protosphyraena* Leidy. 1856. Newton. Q. J. G. S., 1878, p. 787, and has been placed in a new family, *Protosphyraenidae* Woodward. Cope's objection to *Protosphyraena* was that Dr. Leidy did not sufficiently characterize his genus, and for this reason it should be ignored If such objections are deemed valid by naturalists, very many of the genera and species of early paleontologist would cease to be recognized, not only among the fishes, but in every other class of the animal kingdom.

†l. c. 189.

‡Q. J. G. S., Vol. XXIII, p. 507

Further in his modified description of *H. lewesiensis* Agass., Mr. Newton says*:

"I am far from being convinced of the propriety of placing it (*H. lewesiensis*) in the group of the *Saurodontidæ.*"

From the above it would seem that there is some doubt as to the exact systematic position of *Hypsodon*, but until more complete specimens are found, showing the other cranial characters not known at present, it will have to remain in the *Saurodontidæ.*

After having made a careful study of the Saurodont material in the Kansas University Museum, and in view of some new characters which have been brought to light since Prof. Cope made his synopsis, I deem it advisable to divide this family into two groups which I will characterize as follows:

GROUP I.

Carnivorous fishes, many of large size. Jaws without foramina below the alveolar border internally. Teeth cylindric, no predentary. Supra-occipital produced upward into a crest. New crown rises within the pulp cavity of the functional tooth in the succession of the teeth. Embracing the genera, *Portheus*, *Ichthyodectes* and *Hypsodon*, of which *Portheus* should be the type.

GROUP II.

Carnivorous fishes not attaining as large a size as in the first group. Jaws with foramina or deep notches below the alveolar border internally. Teeth compressed, knife-like, or sub-cylindric. Predentary present. Supra-occipital not raised into a crest. Succeeding crown developed outside of the functional tooth as in many *Lacertilia* including the *Mosasaurs.* In this Group are *Daptinus*, *Saurodon*, and *Saurocephalus* of which *Daptinus* should be the type.

Below we give a revised synopsis of the family:

I. Predentary not present, no foramina below the alveolar border internally; teeth cylindric.
  - Teeth of unequal length, some of them are greatly developed .... *Portheus.*
  - Teeth of equal length .... *Ichthyodectes.*

II. Predentary present.
  - a. Foramina occurring below the alevolar border internally.
    - Teeth with short, compressed crowns ...... *Saurocephalus.*
    - Teeth with sub-cylindric crowns .... *Saurodon.*
  - aa. Deep notches occurring below the alveolar border internally .... *Daptinus.*

*l. c., 508.

Portheus Lowii sp. nov.

This species is based upon the dentary bones of a single individual. They were found at Fairbury, Nebraska, in the same horizon of the Fort Benton, with *Desmatochelys lowii* Williston and were sent to the museum by Mr. M. A. Low of Topeka in whose honor the species is named. Special interest is attached to this species as it is the first time the genus *Portheus* has been reported from so low a horizon as the Fort Benton.

The dentary is short with a symphysis more oblique than in any other species of *Portheus* which I have examined. It is also not so roughly marked for the attachment of the ligaments binding the jaws together as in *Portheus molossus*. The alveolar border is shorter and not so thick proportionally as in this species. The groove for Meckel's cartilage is very shallow and the swelling of the alveolar border just back of the symphysis is but slightly developed. The posterior extremity of the alveolar border is projected upward into a short coronoid process, which is but slightly bent outward. The teeth are slightly oval in cross section, and non-striate. At the extremities the crowns are acutely pointed and curved slightly backward. The arrangement of the teeth is as follows: one large, two small, one large, and ten or eleven medium large and small.

Measurements are as follows:

	MM.
Length of alveolar border	177
Length of symphysis	79.5
Depth of dentary at middle	64.5
Depth of dentary just back of symphysis	65

Daptinus broadheadi, sp. nov.

Established on the left superior maxillary and one of the predentaries. The remains were found in Wallace county, Kansas, by Mr. Geo. W. Cooper. Named in honor of Prof. G. C. Broadhead of Columbia, Mo.

The maxillary is less elongate and ends more abruptly than in *Saurocephalus*. The anterior border slopes forward more obliquely than in *Portheus*. The premaxillary surface is continuous with the outer surface of the maxillary, which surface is provided with small tubercular protuberances probably fitting into corresponding depressions on the premaxillary. It is seen from the above that the premaxillary is not so immovably fixed as in *Portheus*, where the premaxillary fits into a deep depression of the maxillary and has a thin lamina of bone extending forward nearly to the extremity supporting it. The ramus is thin above and thickens but slightly at the alveolar border. The bone does not materially thicken

below the palatine condyles as in *Portheus*. The two superior condyles are situated much nearer each other than in the form just mentioned; the anterior is elevated upon a pedicel and is rather tubercular. The palatine is elongate, narrow and nearly flat, it does not have the prominent internal notch which I have observed in *D. phlebotomus*. The teeth are closely set, with compressed knife-like crowns and smooth enamel surface, appearing very slightly striate under the microscope. One deep notch occurs to each tooth; as in the form just mentioned above, alveola for thirty-one are found. A single predentary was found on the same slab with the above, and no doubt belongs to this specimen. It is a small, triangular element, the posterior surface of which is very irregular for cartilage, binding it to the dentary. The upper border is edentulous, the lower thin and sharp; the two borders meet at an acute point anteriorly.

MEASUREMENTS.	MM.
Length of maxillary*	122
Length of palatine condyle	18
Depth of maxillary at the centre	37
Depth of bone at palatine condyle	44
Number of teeth in 1 cm.	3.5
Length of crown measured externally	3.5
Anterior-posterior breadth of crown	2.75
Depth of predentary	28.5
Length of predentary, approximally	30

Saurocephalus dentatus, sp. nov.

Established upon the left maxillary, premaxillary, and mandible of one individual and the left mandible of another. The specimen is from the Niobrara Cretaceous of Wallace county, and was found by Mr. E. P. West.

The maxillary is larger and more elongate than in *Daptinus broadheadi* just described. The superior border is very thin and more elevated just back of the palatine condyle than in *Daptinus*. The palatine condyle is strongly convex from before backward. Anterior to the palatine there seem to have been two condyles which were probably for the ethmoid and vomer, the most posterior of these is broken away but from the base it appears to have been elevated as in *D. broadheadi*. The anterior of these condyles is rather large and triangular in outline and is bounded in front by a shallow pit not found in *Daptinus*. The teeth are similar to those described by Cope.† They decrease in size toward the posterior extremity. Alveola for thirty-eight are found.

*Estimated.
†Cret. Vert., 216.

The premaxillary is more or less plate-like, externally it is convex from before backward. The anterior border is quite oblique and forms an acute angle with the alveolar border. There is probably no close connection with its fellow on the opposite side. The upper portion of the bone is covered with fine lines radiating upward and backward from the tip. The teeth seem to be somewhat smaller than those on the maxillary; alveola for nine are found.

The ramus of the mandible decreases more in depth toward the symphysis than in either *Ichthyodectes* or *Portheus*, the lower portion is very thin, becoming gradually thicker towards the alveolar border but does not attain the robustness of this portion in *Portheus*. Just back of the predental surface and below the line of foramina occurring opposite the roots of the teeth there is a prominent swelling more strongly marked than in *Daptinus*. The predental surface is almost vertical and is very irregular for cartilage attaching it to the predentary. Just back of this and below the swelling mentioned above, internally, there is an elongated ovoid pit near the point of the Mento-Meckelian ossicle of *Amia* The posterior portion of the dentary is well elevated above the articular. The groove for Meckel's cartilage is not so deep as in *Portheus*. From the center to the anterior extremity the teeth decrease in size, on the posterior portion they are about twice as large as those on the superior maxillary: the crowns are compressed and appear minutely striate under the microscope. Just beneath the dentary there is a long, thin element extending nearly its whole length, which appears to be joined to it by a suture. If this be true it may represent a new element in the jaw, although more material will have to be brought to light before this point can be determined.

The predentary is a triangular element joined to the dentary by a very irregular surface broader above than below. The superior borderis finely rugose and edentulous. The tip is acute. The two rami were probably united by ligaments at the symphysis, as in the *Mosasaurs*.

The articular sends a long dagger-like element forward internally nearly to the ovoid pit mentioned above. Externally it is soon covered by the dentary. The cotyloid cavity has its surface more vertically directed than in either *Portheus* or *Ichthyodectes*; it is narrower laterally and slightly concave from above downward. Externally a lamina of bone extends backward probably articulating with the angular below.

MEASUREMENTS.

	MM.
Length of maxillary and premaxillary	161.5
Depth of bone posterior to palatine condyle	44.5
Height of palatine condyle above the alveolar border	48.5
Length of premaxillary, inferior	31.5
Average height of crown	3.9
Average anterior posterior length of crown	3.3

MANDIBLE.

Length of mandible from cotyloid cavity	161
Length of alveolar border	140
Average height of crown, posterior	6
Average anterior posterior length of crown, posterior	4.4
Depth of predental surface*	33
Vertical depth of condyle	13
Predentary, length	29.5
Predentary, depth	34

Protosphyræna bentonia, sp. nov.

Established upon the rostrum, and numerous fragments of bones whose identity cannot be determined. These were found by Dr. S. W. Williston in the Lincoln Marble on Rock creek in southern Mitchell county. The low horizon from which the specimen was obtained attaches special interest to it as it is the first species described from below the Niobrara Cretaceous.

The proximal portion bearing the larger ethmoidal teeth is not preserved. The base is broad, becoming more narrow toward the distal extremity, where it suddenly contracts, forming a rather blunt apex. In *P. penetrans* the bone gradually contracts to an acute point. The anterior portion is oval in outline instead of semicircular as in the above, nor does it have the flat superior surface of this species as described by Cope.† The lower surface contracts more rapidly than the upper, causing the apex to be above the center of the shaft. The inferior and superior surfaces gradually grade into each other laterally and are not separated by the obtuse angular ridge found in *P. xiphoides*. The outer surface, where preserved, shows the rostrum to be covered with irregular longitudinal ridges which send out branches to each other, and giving the outer surface of the bone the appearance of net work. The other bones are too fragmentary to identify.

*Estimated.
†Bull. U. S. Geol. Surv. Terr., Vol. III, No. 4, p. 822.

MEASUREMENTS.

	MM
Length of rostrum, fragment	199
Transverse diameter 136 mm. from anterior extremity	34
Transverse diameter 22 mm. from anterior extremity	22

Protosphyræna, sp. nov,

Established upon a right premaxillary from the Niobrara Cretaceous, the exact locality not known. The material upon which this species is based is too scanty I think to justify a specific name being given to it until more complete specimens are found. It may prove to be a synonym of *P. penetrans* or *P. xiphoides* when specimens of these are found with the premaxillary attached, but for the present at least it will have to regarded as a new species.

When viewed from the side, the bone is triangular in outline with a thin superior and posterior border. The anterior extremity is broken away but was probably acutely pointed as in *P. nitida.* The difference which characterizes this form from *P. nitida* are the size and arrangement of the teeth. Near the center of the bone there are alveola for four large teeth, the three anterior of which are preserved nearly complete, the posterior of these is broken off at the base but it and the anterior one seem to be the smallest of the four. The two in the center are of about the same size; they are all directed forward, the anterior slightly more than the rest. These teeth all have broad lanciform crowns with sharp anterior and posterior cutting edges and slightly striate enameled surfaces. Just back of the most posterior of these there is a row of small teeth, the anterior of which are hardly more than bony tubercles but posteriorly they assume definite dental characters; ten of these are present in the specimen. There are other teeth like these present on the anterior portion of the border, the exact number of which cannot be determined, owing to the tip of the bone being broken away. The maxillary surface contracts toward the anterior, and is bounded below by a narrow shelf of bone extending inward.

MEASUREMENTS.

Approximate length of the alveolar border	82
Depth just back of last large tooth	27
Height of first large tooth	15
Height of second large tooth	19
Length of first large tooth	8
Length of second large tooth	9

The species of *Protosphyræna* now known are as follows:

	Niobrara Cretaceous.	Fort Benton Cretaceous.
American Cretaceous:	*P. nitida* Cope (*P. angulatus* Cope).	*P. bentoniana* Stewart.
	P. penetrans Cope.	
	P. species, Stewart.	

English Cretaceous: *P. ferox* Leidy (*Erisichthe dixoni* Cope, *Xiphias dixoni* Leidy).

Additional treatment will probably be given to this subject in the report on the Cretaceous Fishes of Kansas now in course of preparation by the author.

Alternating Currents in Wheatstone's Bridge.

BY M. E. RICE.

For the solution of problems involving continuous currents in a net work of conductors, two general laws suffice, viz. Kirchhoff's Laws which are, (1) In any net work of conductors the algebraic sum of all the currents flowing to or from a junction is zero. (2) The sum of all the E. M. F.'s in a closed circuit equals zero if the E. M. F. consumed by resistance, IR, is also considered as a counter E. M. F., and all the E. M. F.'s are taken in their proper direction.

But when the corresponding problem involving alternating currents is met with, these laws do not apply except to the instantaneous values. Hence, in general, the solution of such problems requires the solving of several differential equations more or less involved; these equations often being too complicated for solution except in special cases.

This lack of generality in the application of Kirchhoff's laws has been overcome by Mr. C. P. Steinmetz who has shown *that if the algebra of the plane instead of that of the straight line be employed, the laws are entirely general.

In this method, electromotive forces, currents, and impedances, (corresponding to resistances for continuous currents), are all expressed as complex quantities, e. g.; $a+jb$, where $j=\sqrt{-1}$. Thus the absolute value of the quantity, its modulus, is $\sqrt{a^2+b^2}$ and its phase angle, its amplitude, is $\tan^{-1}\frac{b}{a}$.

Kirchhoff's laws may accordingly be written:

(1) At a junction point $\Sigma I=o$.

(2) In a closed circuit $\Sigma IZ=\Sigma E$,

$$\text{where } E=e+je',\ |E|=\sqrt{e^2+e'^2}$$

$$I=i+ji',\ |I|=\sqrt{i^2+i'^2}$$

$$Z=r-jx,\ |Z|=\sqrt{r^2+x^2}$$

*See Proceedings of the International Electrical Congress at Chicago, 1893, pp 33-75; also "Alternating Current Phenomena," Steinmetz, issued in 1897.

The large letters represent complex quantities or vectors and the small letters denote real or scalar quantities: r denotes the resistance and x the reactance of a branch, i. e. $x = L\omega - \frac{1}{C\omega}$ where ω is 2 times the frequency of alternation.

It is the purpose of this paper to apply the method outlined above to the solution of problems involving alternating currents in the branches of a Wheatstone's Bridge.

Let the branches of a Wheatstone's Bridge be represented in Fig. 1, where the branches are numbered (1), (2), ... (6), and the arrows indicate assumed instantaneous directions of currents and electromotive forces. Branch (5) contains the galvanometer or telephone, and (6) the impressed E. M. F; a battery, to be closed after the galvanometer circuit is closed; or the secondary of an induction coil, with telephone in (5).

Kirchhoff's laws give the six equations:

$$
\begin{aligned}
I_1+I_2-I_6&=0\\
I_1-I_3-I_5&=0\\
I_3+I_4-I_6&=0\\
Z_3I_3-Z_4I_4-Z_5I_5&=0\\
Z_1I_1-Z_2I_2+Z_5I_5&=0\\
Z_2I_2+Z_4I_4+Z_6I_6&=E_6
\end{aligned}
\qquad [1]
$$

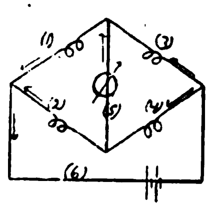
The condition for no current in the galvanometer is $I_5=0$, which gives

$$
I_5 = \frac{\begin{vmatrix} 1 & 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & z_3 & -z_4 & 0 & 0 \\ z_1 & -z_2 & 0 & 0 & 0 & 0 \\ 0 & z_2 & 0 & z_4 & E_6 & z_6 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & z_3 & -z_4 & -z_5 & 0 \\ z_1 & -z_2 & 0 & 0 & z_5 & 0 \\ 0 & z_2 & 0 & z_4 & 0 & z_6 \end{vmatrix}} = \frac{E_6(z_2z_3-z_1z_4)}{\cdots} = 0 \qquad [2]
$$

This reduces at once to the very simple equation,

$$
z_2z_3 = z_1z_4 \qquad [3]
$$

Fig. 1.



Suppose first that the arms of the bridge contain only resistances. Then

$$z_1=r_1 \quad z_3=r_3$$

$$z_2=r_2 \quad z_4=r_4$$

and equation [3] becomes $r_2r_3=r_1r_4$, [4]

the same as for continuous currents.

Suppose next that each arm of the bridge contains a resistance and an inductance, then $z_1=r_1-jx_1$, $z_2=r_2-jx_2$, $z_3=r_3-jx_3$, $z_4=r_4-jx_4$, and equation [3] becomes

$$r_2r_3-x_2x_3-j(r_3x_2+r_2x_3)=r_1r_4-x_1x_4-j(r_1x_4+r_4x_1) \qquad [5]$$

Transposing, and equating the reals to zero and the imaginaries to zero, gives the two equations of condition

$$r_2r_3-r_1r_4=x_2x_3-x_1x_4 \qquad [6]$$

$$r_1x_4+r_4x_1=r_3x_2+r_2x_3 \qquad [7]$$

Hence, in general, if any six of the constants of the bridge be given, the other two may be obtained from equations [6] and [7]. For example, if the resistances of three arms be given, the necessary resistance and inductance of the fourth arm my be calculated. Or, if the four resistances and two of the inductances are known, the other two inductances may be calculated.

It is evident from equation [6] that the bridge need not be balanced for continuous currents in order to be balanced for alternating currents. But if the bridge is first balanced for continuous currents, equations [6] and [7] reduce to

$$x_1x_4=x_2x_3 \qquad [8]$$

$$\frac{x_4}{r_4}+\frac{x_1}{r_1}=\frac{x_3}{r_3}+\frac{x_2}{r_2} \qquad [6]$$

In this case also, if two of the inductances, say x_1 and x_2, are given, one and only one pair of values of x_3 and x_4 can be obtained that will balance the bridge. But if $\frac{x_1}{x_2}=\frac{r_1}{r_2}$, equation [9] reduces to $\frac{x_3}{x_4}=\frac{r_3}{r_4}=\frac{r_1}{r_2}$ which is the same as [8]; consequently in this case the ratio $\frac{x_3}{x_4}$ only can be determined from [8] and [9], and

there is an indefinite number of pairs of valves of x_3 and x_4 that will balance the bridge, and if one inductance, say x_3, is known, the other is readily found.

To compare two self-inductances make $x_3 = x_4 = 0$ in equations [6] and [7], giving

$$r_2 r_3 = r_1 r_4 \qquad [10]$$

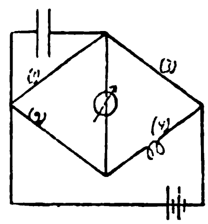
$$r_4 x_1 = r_3 x_2 \qquad [11]$$

That is, the bridge is first balanced for continuous currents, then $\frac{x_1}{x_2} = \frac{r_3}{r_4}$ and the comparison is at once made. This is Maxwell's method.

To compare a self-inductance with an electrostatic capacity, arrange the bridge as in Fig. 2. In this case the impedances of equation [3] are

$$z_4 = r_4 - jx_4$$
$$z_3 = r_3$$
$$z_2 = r_2$$
$$z_1 = \frac{x_c r_1}{x_c - jr_1}.$$

Fig. 2.



Z_1 is obtained thus:

$$\frac{1}{z_1} = \frac{1}{r_1} + \frac{1}{jx_c} = \frac{x_c - jr_1}{x_c r_1}$$

Where $x_c = \frac{1}{C\omega}$ C being the capacity of the condenser connected in parallel with branch (1).

Substituting these values in [3] gives

$$\frac{(r_4 - jx_4) x_c r_1}{x_c - jr_1} = r_2 r_3$$

which gives the two equations

$$r_1 r_4 = r_2 r_3 \qquad [12]$$

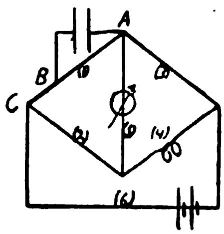
$$x_4 x_c = r_2 r_3 \qquad [13]$$

Here $x_4 = L\omega$ and $x_c = \frac{1}{C\omega}$ and [13] reduces to $L = Cr_1 r_4$. the form given by Maxwell.

A better practical arrangement of parts is shown in Fig. 3, where

the condenser is shunted around only a part of the resistance in branch (1). If the resistance A to C$=r_1$ and A to B$=r'_1$ then

Fig. 3



$$z_4=r_4-jx_4$$

$$z_3=r_3$$

$$z_2=r_2$$

$$z_1=r_1-r'_1+\frac{x_c\, r'_1}{x_c-jr'_1}=\frac{x_c\, r_1-jr'_1(r_1-r'_1)}{x_c-jr'_1}.$$

Substituting these values in equation [3] gives the two conditions

$$x_c(r_2r_3-r_1r_4)=x_4(r'^2_1-r_1r'_1) \qquad [14]$$

$$r_1x_4x_c=r_1(r_2r_3-r_1r_4)+r'^2_1r_4 \qquad [15]$$

And if the bridge is first balanced for continuous currents, equation [15] reduces to

$$x_4=\frac{r'^2_1 r_4}{x_c\, r_1},$$

or putting in the values of x_4 and x_c the relation becomes

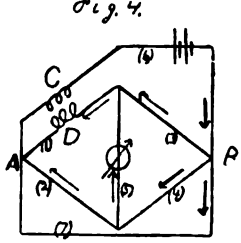
$$L=\frac{Cr'^2_1\, r_4}{r_1},$$

the one given by Rimington. In this case the bridge is first balanced for steady currents and then the point B is found so as to balance for variable currents, without repeated adjustments of resistances.

To compare the mutual inductance of two coils, C and D, with the self-inductance of one of them, D, consider the arrangement indicated in Fig. 4, a Wheatstone's bridge with an extra conductor from A to B. Kirchhoff's laws give the seven equations:

$$\begin{aligned}
&I_1+I_2-I_6+I_7=o\\
&I_1-I_3-I_5=o\\
&I_3+I_4-I_6+I_7=o\\
&Z_3I_3-Z_4I_5-Z_5I_5=o \qquad [16]\\
&Z_1I_1-Z_2I_2+Z_5I_5=jx_m\, I_6\\
&Z_2I_2+Z_4I_4+Z_6I_6=E+jx_m\, I_1\\
&Z_2I_2+Z_4I_4-Z_7I_7=o
\end{aligned}$$

Fig. 4.



The condition for $I_5=o$, derived as in the preceding problems, is

$$(z_2z_3-z_1z_4)z_7+jx_m(z_2z_3+z_3z_4+z_3z_7+z_4z_7)=0. \qquad [17]$$

The values of the various impedances in this case are:

$$z_1=r_1-jx_1 \qquad z_5=r_5-jx_5$$
$$z_2=r_2 \qquad z_6=r_6-jx_6$$
$$z_3=r_3 \qquad z_7=r_7$$
$$z_4=r_4$$

which substituted in equation [17] reduce it to

$$(r_2r_3-(r_1-jx_1)r_4)r_7+jx_m(r_2r_3+r_3r_4+r_3r_7+r_4r_7)=0.$$

Equating the reals to zero and the imaginaries to zero gives

(1) $$r_2r_3=r_1r_4$$

(2) $$x_1r_4r_7+x_m(r_2r_3+r_3r_4+r_3r_7+r_4r_7)=0$$

or, combining (2) with the first condition,

$$x_1=-x_m\left(1+\frac{r_1}{r_2}+\frac{r_1+r_3}{r_7}\right), \qquad [18]$$

where $x_1=L_1\omega$ and $x_m=M\omega$.

If the branch [7] is omitted, [18] reduces to the simpler form

$$x_1=-x_m\left(1+\frac{r_1}{r_2}\right). \qquad [19]$$

Both [18] and [19] are given by Maxwell, the latter being the simpler case theoretically, the former the easier of practical application.

The above are only a few of the problems that can profitably be attacked by this method; but they are sufficient to show its great generality and ease of application.

Adulterations of Buckwheat Flour Sold in the Lawrence Market.

BY MARSHALL A. BARBER.

With Plates III and IV.

Seven samples of buckwheat flour were purchased of grocers in Lawrence, Kansas. No two samples were obtained of any one dealer, and the purpose for which they were bought was not given. One variety was said to come from Michigan, one from New York, and one from Tennessee; while the other four were from different mills in Douglas and Leavenworth counties. Inquiry was made in nearly every case as to the purity of the flour, and the purchaser was always assured that the sample was "pure buckwheat."

The examination was made with the compound microscope, and the results are best explained by the photomicrographs illustrating this article. In these the objects are magnified about 265 diameters.

Fig. 1 is from a photomicrograph of pure buckwheat. The sample was obtained by grinding in the laboratory buckwheat free from all other grains, so that a flour, known to be genuine, was at hand for comparison. The two fragments shown fairly represent the masses of starch grains seen in buckwheat flour. These grains vary comparatively little in size, they are closely compacted in the masses, and the individual grains are somewhat angular in outline and show few concentric lines.

Fig. 2 represents a mass of starch of wheat obtained from shorts. In the wheat flour fragments the starch grains vary much in form, and the larger ones far exceed in size the largest of the buckwheat starch grains. Their outline is more regular than that of the buckwheat grains, and the prevailing forms are round and elliptical. With comparatively little magnification the wheat starch grain is shown to have well marked concentric lines, and to differ from the buckwheat in the form of its center. These two points are not clearly shown in the reproductions of the photomicrographs; but the comparative uniformity in size of the buckwheat starch grains and their smallness and irregularity of form make the fragments of this flour easily distinguishable from those of wheat.

Fig. 3 represents a sample of a "pure buckwheat" of the "instantaneous rising" kind. Two masses of buckwheat starch are shown, and with them a somewhat larger amount of another starch very much like the kind seen in shorts. The proportion between the buckwheat and the other starch in this flour is fairly well represented by this figure, and no more than half of the sample examined was buckwheat.

Figs. 4, 5 and 6 are of samples, each from a different mill, and all were adulterated. The proportion of buckwheat in each was two-thirds or more; so that the photomicrographs give, in some cases, too great, in others, too little buckwheat in proportion.

The samples from Michigan, New York and Tennessee were pure, or practically pure; and, since they were essentially like the sample represented by Fig. 1, no illustrations of them are given. In the Michigan sample a few grains of wheat starch were found, but so few that it is not likely that they were intentionally added. The adulterated samples all came from mills in Douglas and Leavenworth counties. In three of them there was, as stated above, one-third or less of the adulterant; in one the flour added formed nearly one-half of the mixture.

In every case a second sample taken from a different part of the package purchased was examined, and the results of the first examination were confirmed. The adulterant in these samples closely resembles wheat starch. Certain grades of shorts are said to be often used to adulterate buckwheat, and this is possibly the source of the starch in this adulterant. Kaffir corn flour is also said to be used in some parts of this state as an adulterant of buckwheat; neither this, nor, besides wheat, any other flour likely to be used for mixing with buckwheat was found in these samples, though a comparison with various starches was made. So the evidence is good that wheat starch of some grade was the adulterant used here.

It is doubtless true that buckwheat flour is made more wholesome and palatable by the addition of a certain proportion of some other flour; but it would seem fairer to the purchaser to let him know how much other flour is mixed in, rather than to label the package "Pure Buckwheat," or otherwise represent it as such. Some, perhaps, might not like so large a quantity of wheat flour in their buckwheat as that found in the sample represented in Fig. 3; and all, probably, would prefer that the price of the article diminish as the proportion of cheaper flour increases.

Editorial Notes.

Dr. George I. Adams has published in the American Journal of Science for 1897 a short article on Extinct Felidæ.

Mr. Paul Wilkinson has published in the Transactions of the American Institute of Mining Engineers an article on The Technology of Cement Plaster.

Professor H. B. Newson has in a recent Bulletin of the American Mathematical Society, December, 1897, an article on Continuous Groups of Circular Transformations.

The *Industrialist*, from the Kansas State Agricultural College, appears in magazine form with the first issue of the year 1898. While retaining something of its former character as a local bulletin and reporter for the college, the *Industrialist* will henceforth serve in the Agricultural College the same function as the QUARTERLY in the State University. The institution is to be congratulated on the achievement and its promise.

Dr. George O. Virtue has in the U. S Bulletin of the Department of Labor for November an article on The Anthracite Mine Laborers. Dr. Virtue has made a study of the whole anthracite industry.

Ueber den Hermite'schen Fall der Lamischen Differentialgleichung. Inaugural-Dissertation zur Erlangung der Doktorwuerde der hohen Philosophischen Facultaet der Georg-Augusts Universitaet zu Goettingen, vorgelegt von Mary Frances Winston aus Chicago. Goettingen, 1897, pp. 84 and 32 plates.

This publication by Miss Winston, the new professor of Mathematics in the Kansas State Agricultural College, is in all respects worthy of that lady's reputation as a mathematician. It contains a detailed and exhaustive study of one of the most important differential equations arising in mathematical physics and it is thus a substantial contribution to the world's stock of useful knowledge. This thesis contains besides a theoretical discussion of the equation an application of the results to the mechanics of the spherical pendulum and to the theory of the top.

Lame's differential equation

$$\frac{d^2y}{dt^2}=\{Ap(t)+B\}y \text{ where } A=n\,(n+1);\ (n \text{ is any integer}) \text{ and } P(t)=x.$$

is linear of the second order and first appears in connection with the problem of heat conduction in a solid body. The integration of this equation has taxed the ingenuity of a generation of mathematicians. In 1874 the now venerable Hermite of Paris published his solution by means of elliptic functions. Hermite's results are complicated formulae which render it possible to compute the values of y for any given value of x. In order to grasp the significance of Hermite's solution it is necessary to have a geometrical representation of the results reached. Miss Winston has plotted the real curves representing the integrals for many special cases and these curves are here reproduced in 32 costly plates.

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delivered a course of lectures before the American Mathematical Society; he chose for his subject the Theory of the Top. One of the most recent works from the press of B. G. Tuebner of Leipsic is the first part of a treatise on the same subject by Prof. Klein. This dissertation was suggested by Klein and developed under his guiding hand. This is sufficient assurance that the work is up to date. That it possesses real merit is also vouched for by the fact that this thesis won for its author from the University of Goettingen the degree of Doctor of Philosophy, *magna cum laude*.

Although this work was written before its author became a Kansan, it reflects credit upon the state and the state institution which she represents.—H. B. N.

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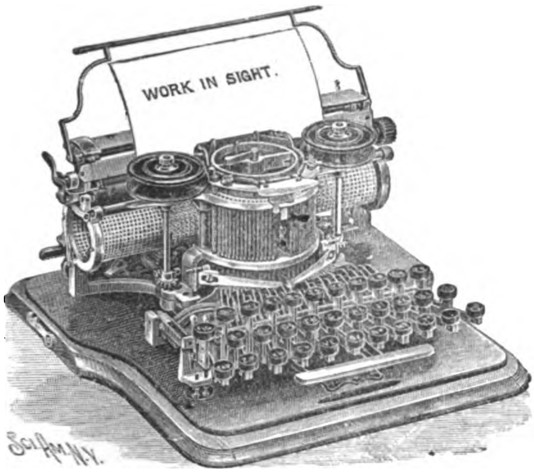

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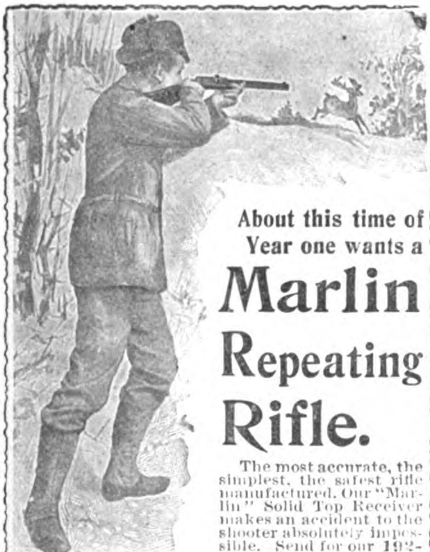

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KANSAS UNIVERSITY QUARTERLY.

VOL. VII. APRIL, 1898. No. 2.

The Designing of Cone Pulleys.

BY WALTER K. PALMER.

INTRODUCTION.

Probably no other minor operation of machine designing involves such a complex mathematical analysis as the apparently simple one of proportioning a pair of cone pulleys.

The question of determining a single pair of pulleys to give a desired velocity ratio for their shafts, is so elementary and so easily solved by the simplest application of arithmetic, that it is difficult at first to realize that there can be anything at all to make impossible an equally simple treatment of the problem of a series of pulleys on one shaft, paired to run with a corresponding series on another, which constitute what are commonly called "cone," or "step cone" pulleys—the several pulleys of each being the "steps."

But the one condition which must be observed when proportioning the series of steps of a cone pulley, and which is not imposed in the case of a series of independent pairs of pulleys—that the *same belt* must fit with an equal degree of tension on each one of the pairs of the series—introduces complications which make the problem a most difficult one for exact solution; and one which, it is believed, has not yet been treated in full by an exact method, either analytical or graphical. At least no method, not involving some kind of an approximation, or tentative process, has been offered, which is of a satisfactory form to use in the course of every day practice.

A purely analytical solution is not to be expected, owing to the form of the equations, as will appear. And such a solution is not

so much to be desired as just the right kind of a graphical construction. The form of this construction should be so simple that it can be drawn at once from memory for any particular case in hand, and with the simple drafting instruments, without the use of the irregular curve or any special appliance. And this construction must be so complete as to give *all* of the desirable features of a solution for every possible case of the problem, directly and with exactness. And it should be based on a non-approximate analysis.

These requirements are rigid ones, and ones which the general equation for the problem would seem to offer little promise of ever fulfilling. But it has seemed so highly desirable that there should be a solution conforming exactly to each of the restrictions named, that the question has been studied from every point of view in the determination to find a treatment which would not be a compromise in any particular, if such a treatment were in any way possible.

As the result, the present discussion of the problem is offered, with the deduction of a practical method, which, it is believed, embodies all of the desirable features at first determined upon.

THE PROBLEM.

The following is the problem: We should be able,

I. To assume any distance between centers of the shafts;

II. To choose the radius of a step on one shaft, and to get the radius of the corresponding step on the other, the two radii to be in a predetermined ratio. This can be done arithmetically, but should be included in the graphic process.

III. From this pair we should be able to have, at once, the length of belt required for the two cones. This length of belt is now a constant quantity, and must fit all the other pairs of steps about to be determined.

IV. Dependent upon this length of belt, we should now be able to obtain readily an indefinite number of pairs of radii, which will "run" to this determined length of belt with the same degree of tension.

V. From all these possible pairs which run to this length of belt, we should be able to select a certain pair of radii, which shall bear a definite ratio to each other.

In addition, we have from practical considerations that the series of speeds should form a geometrical progression, except in cases where particular reasons exist for having a certain definite speed at each step. This geometrical series of speeds is readily obtained if condition V is fulfilled.

Bearing in mind these five specific conditions, and the requirements as to the character of the solution desired, as already discussed, we may proceed to analyze the problem and derive a satisfactory treatment.

GENERAL ANALYSIS.

There are two general cases of the problem: I. Open Belts; II. Crossed Belts.

CASE I—OPEN BELTS.

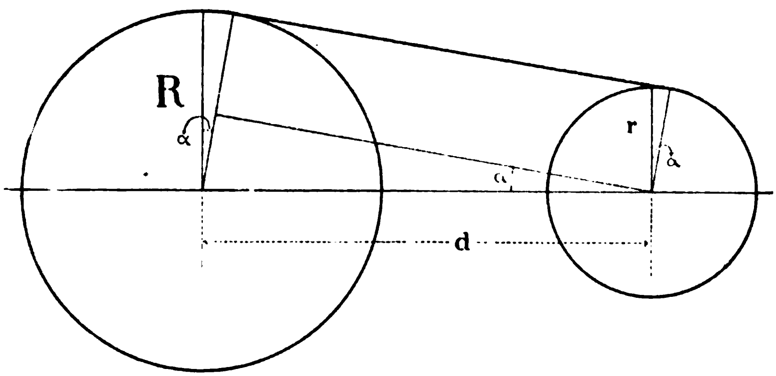

Fig. 1.

Let l=half length of belt.
d=distance between centers of shafts.
R=radius of pulley on one shaft.
r=radius of pulley on the other.
α="angle of the belt," as shown in Fig. 1.

From the geometry of the figure we have

$$l=\frac{\pi}{2}R+\alpha R+d\cos\alpha+\frac{\pi}{2}r-\alpha r,$$

from which

$$l=d\cos\alpha+\frac{\pi}{2}(R+r)+\alpha(R-r). \qquad (1)$$

And

$$\alpha=\frac{l-d\cos\alpha-\frac{\pi}{2}(R+r)}{R-r}.$$

From the figure

$$\sin\alpha=\frac{R-r}{d}$$

and

$$\cos\alpha=\frac{\sqrt{d^2-(R-r)^2}}{d}.$$

$$\therefore \quad \frac{R-r}{d} = \sin\alpha = \sin\left(\frac{l-\sqrt{d^2-(R-r)^2}-\frac{\pi}{2}(R+r)}{R-r}\right) \qquad (2)$$

This is the equation for the relation between the two radii, length of belt, and distance between the centers of the shafts. It is transcendental, and plainly of such a form as to be of no value for direct use in working to the desired result. It serves merely to show the relation existing between these quantities, and the difficulty of attaining the desired form of solution.

CASE II—CROSSED BELTS.

Fig. 2 shows the other general case, that of crossed belts.

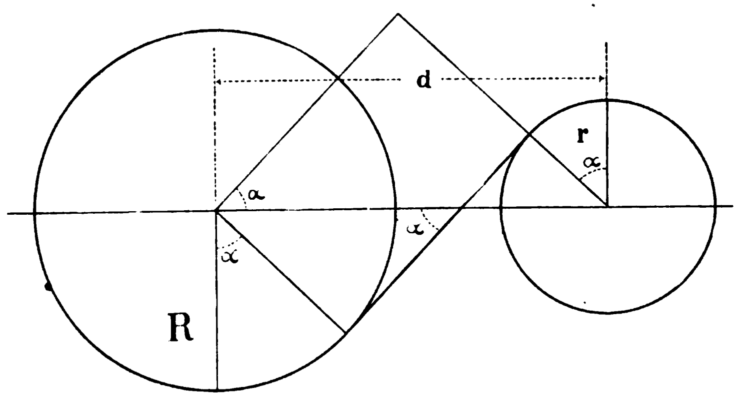

Fig. 2.

Treating this in the same way as the other case, we have for the half length of belt:

$$l = \frac{\pi}{2}R + \alpha R + d\cos\alpha + \frac{\pi}{2}r + \alpha r,$$

the sign of the last term being positive instead of negative as before. And, here,

$$\sin\alpha = \frac{R+r}{d}$$

and

$$\cos\alpha = \frac{1}{d}\sqrt{d^2-(R+r)^2}.$$

From which we have

$$l = d\cos\alpha + \left(\frac{\pi}{2}+\alpha\right)(R+r); \qquad (3)$$

$$\therefore \frac{\pi}{2}+a=\frac{l-d\cos a}{(R+r)},$$

and

$$\cos\left(\frac{\pi}{2}+a\right)=\cos\left(\frac{l-d\cos a}{R+r}\right);$$

$$\therefore \sin a=\cos\left(\frac{l-d\cos a}{R+r}\right)=\frac{R+r}{d},$$

$$\therefore \frac{R+r}{d}=\cos\left(\frac{l-\sqrt{d^2-(R+r)^2}}{R+r}\right). \tag{4}$$

An inspection of equations (3) and (4), and Fig. 2, shows that this case of crossed belts does not present the difficulties of the other, and far more important case of open belts. For, if $(R+r)$ be kept constant, a, and hence l, will be constant. This means that we may proportion the first pair of steps of the cones by the simple arithmetical rule, and then any other pair of radii whose sum is equal to that of the first pair will serve for the radii of another pair of steps which the same belt will fit.

The length of belt is not thus given, but as this is of minor importance the proceeding just outlined would answer sufficiently well for all practical purposes. If neither case presented greater difficulties, there would be no occasion for an extended treatment of the problem. But as the case of open belts necessitates a special graphical process, the simpler case of crossed belts will be included in the same method, and a useful diagram analogous to that required for the first case presented.

COMPARISON OF EXISTING METHODS.

As the general relation of equation (2) for the case of open belts offered no possibility of a direct solution of the character required, a comparative investigation was made of the various ways in which the problem has been attacked, and of the character of the methods proposed by leading authorities.

The following is a reference list of some of the best methods heretofore used for treating the problem, with a brief note as to the character of each:

Unwin's "Elements of Machine Design" (new edition), page 373, calls for tedious calculations with approximate formulæ.

Rose's "Modern Machine Shop Practice," gives tables for finding the radii of the steps. Unsatisfactory to use.

Rose's "Complete Practical Machinist," gives a rule for the radius of a circle arc upon which the middle of the steps of the cone will lie, in terms of the length of belt. This necessitates calculating the length of belt for the first, or assumed radii, which is difficult except for cones just alike, with an odd number of steps each. Then when this is done there is no means of obtaining another pair of radii for the same belt, which shall be in predetermined ratio, except tentatively.

Rankine's "Applied Mechanics," page 457, uses approximate equations. Wholly unsatisfactory.

Weisbach, Vol. III, page 262; approximate equations.

Kent's "Mechanical Engineer's Pocket Book," page 874, gives an approximate graphical diagram. Not satisfactory in view of the five requirements discussed. Also gives approximate analytical treatment.

Robinson's "Principles of Mechanism," page 247, an approximate graphical method. Diagram inconvenient in use.

Reuleaux's "Constructor," H. H. Suplee translator, page 189. A *non-approximate* graphical treatment of great interest. The final figure, offered as a permanent working diagram for all cases of cone pulleys, approaches very nearly to an entirely satisfactory form. It, however, embodies two objections: 1st. A permanent diagram, with an irregular curve, does not appear to be as desirable as a simple construction which can be performed at any time from memory for any particular case; 2d. Unless the cones are to be alike, and have an odd number of steps, so that the radii of the middle pair of steps may be the assumed radii, with ratio 1 : 1, it is impossible to find the position of the reference line from which the other radii are to be determined for this length of belt, except tentatively.

Of all the methods examined, however, the graphical treatment of Reuleaux presents by far the greatest possibilities. Following this very closely, with all the desirable features of a complete treatment constantly in mind, it developes that by a slight yet very essential modification at the close of the discussion the desired method may be had in full.

It will be necessary, therefore, to follow the Reuleaux analysis, with no deviation whatever, up to the final step, where, by a special modification of the last figure, the clue to the new method is found.

THE REULEAUX ANALYSIS.

THE DISCUSSION.

The following is the discussion found in the translation of Reuleaux's "Constructor," page 189, essentially as given there, merely amplified somewhat for the sake of clearness. It is remarkable in that instead of attempting to eliminate the "angle of the belt" in some way, as is usually done, this angle is carried on through the discussion and used to a point where it easily disappears. The entire Reuleaux solution shows a wonderful insight into the relations of the problem and extreme ingenuity in dealing with them.

CASE I—OPEN BELTS.

Referring again to Fig. 1 we have for this case equation (1):

$$l = d \cos \alpha + \frac{\pi}{2}(R+r) + \alpha(R-r)$$

and the equation, $d \sin \alpha = R - r$.

Combining these we have the two equations for R and r, respectively:

$$R = \frac{l}{\pi} - \frac{d}{\pi}(\alpha \sin \alpha + \cos \alpha) + \frac{d}{2} \sin \alpha \qquad (5)$$

$$r = \frac{l}{\pi} - \frac{d}{\pi}(\alpha \sin \alpha + \cos \alpha) - \frac{d}{2} \sin \alpha, \qquad (6)$$

which differ from each other only in the sign of the last term.

In Fig. 3 draw AD and BC parallel and at a distance AB apart, equal d of the above equations. Draw AB, leaving the length of the rectangle undetermined as yet. Then draw the quadrant BE, with radius AB$=$d. Now, within the limits of this arc BE will lie all values of angle α, of equations (5) and (6), which can occur. For, from a physical consideration of the matter with the aid of Fig. 1, it is readily seen that α is limited by $0°$ and $90°$. That is, at the limiting case in one direction, when the two pulleys are equal, or diminish to mere points, $\alpha = 0$; and when one decreases to $r=0$ while the other increases to $R=d$, then $\alpha = 90°$. Now let α have any value, as $\angle$ EAP, Fig. 3. Draw PN perpendicular to AP at P, i. e. tangent to the arc BE at P. And make PN$=$arc PE. That is for any value of α, N is to lie on the involute EF, of the arc BE. Drop the perpendicular PM to AD, and draw NK perpendicular to PM. Draw RQ through N, perpendicular to AD.

Then from the geometry of the figure we have

$$AQ = AM + MQ = d(\alpha \sin \alpha + \cos \alpha), \qquad (7)$$

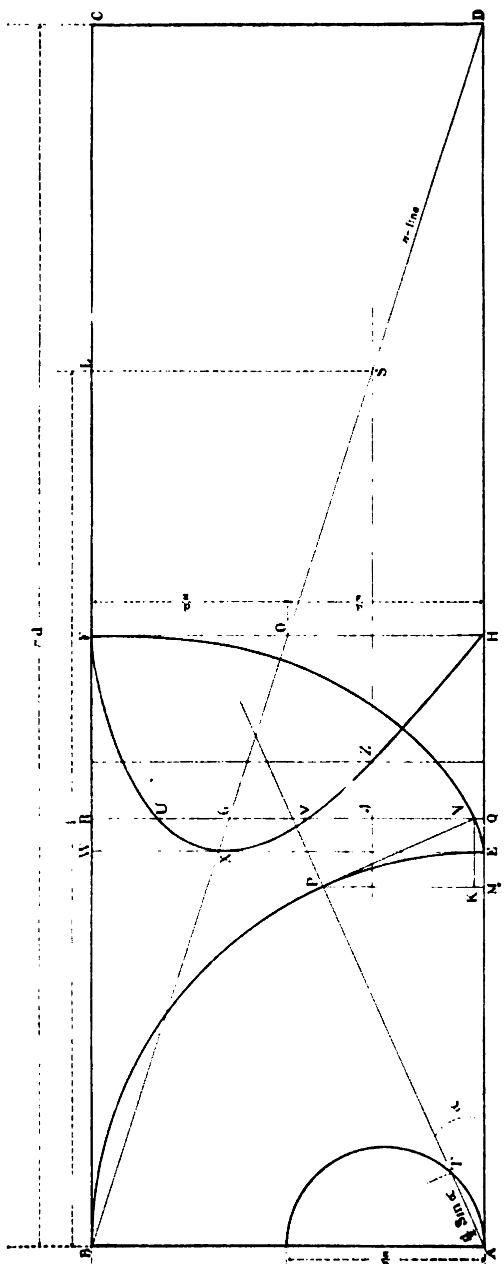

Fig. 3.

which is the middle term of each of the equations, except *not* divided by π.

For, $AM = d \cos \alpha$,

$$MQ = KN = PN \sin \alpha = d\alpha \sin \alpha,$$

$$\therefore \quad AQ = d(\alpha \sin \alpha + \cos \alpha).$$

We have, then, all the values of this middle term (except π times too large) readily obtainable from this involute by varying α. We can now arrange to divide this expression by π for any value α may have, and thus secure the middle terms of equations (5) and (6) complete, as follows:

Take the point F where the involute cuts BC, and drop a perpendicular FH to AD. Assume the middle point O of FH and draw BO, producing it on to its intersection with AD. This line BO cuts RQ, already drawn, in a point G, and GR is equal the desired term

$$\frac{d}{\pi}(\alpha \sin \alpha + \cos \alpha).$$

For $BF = \frac{\pi}{2} . d$, by the property of the involute, and $FO = \frac{d}{2}$ by construction. Then

$$\frac{FO}{BF} = \frac{\frac{d}{2}}{\frac{\pi}{2} . d} = \frac{1}{\pi}.$$

Then by similar triangles

$$\frac{RG}{BR} = \frac{FO}{BF} = \frac{1}{\pi}. \qquad \therefore \; RG = \frac{1}{\pi} . BR.$$

$$\text{But } BR = AQ = d(\alpha \sin \alpha + \cos \alpha).$$

$$\therefore \; RG = \frac{d}{\pi}(\alpha \sin \alpha + \cos \alpha).$$

We notice, now, that the line BO is inclined so that any ordinate drawn down from BC to it is the $\frac{1}{\pi}$ part of the corresponding abscissa, as $GR = \frac{1}{\pi} . BR$. So, then, it is very easy to get the first term of each of the equations (5) and (6) by laying off $BL = l$, the half length of belt, when $LS = \frac{l}{\pi}$, and we have, at once

$$JG = JR - GR = LS - GR$$

$$= \frac{l}{\pi} - \frac{d}{\pi}(\alpha \sin \alpha + \cos \alpha).$$

This is assuming l to be known, when, in reality, it is involved in the other quantities, but this may be done while establishing the relations.

We now have the first two terms of the expression for R and r, shown in the length JG. So if now we add $\frac{d}{2}$ sin α to JG, and then subtract it, we have then the lengths of two lines representing R and r, respectively. This term $\frac{d}{2}$. sin α may be had at once for any value of α by simply drawing a semicircle upon $\frac{1}{2}$ $\overline{AB}$ as diameter, as shown in Fig. 3. The intercept AT of the radius AP, at any position is then $\frac{d}{2}$ sin α.

[This step differs from the corresponding one in Reuleaux's work, the result being the same. The semicircle is more satisfactory, giving at once $\frac{d}{2}$ sin α without drawing any additional lines.]

Lay off AT$=\frac{d}{2}$ sin α upward from G, and downward from G, thus determining points U and V, when we have

$$JU = R,$$

and $$JV = r,$$

l being the correct half length of belt for these radii.

Now similar points may be gotten in just the same way for other values of α. And when all values possible have been used the result will be the smooth curve FUXVH, tangent to WE at X, which gives all possible pairs of radii which run together for any length of belt, the distance between centers of shafts being known or assumed equal d.

THE COMPLETE SOLUTION.

We now have in Fig. 3 a complete solution of the general problem for the cases of open belts, perfectly correct and complete in every detail in so far as the analysis is concerned, though as may be seen not yet of a form suitable for actual use.

Any length of belt as l, in the figure, may be assumed, and the reference line JS quickly found, when the pair of co-ordinates to

this curve FUXVZH give all the possible pairs of radii which will run correctly to this length of belt. For this particular length of belt only the portion of the curve between verticals through X and Z is useful, evidently. This portion gives all possible pairs of radii for this l, while for any other value of l more or less of the curve would be brought into service.

The entire curve would, plainly, come into use only for the theoretical limiting case of one step, $R=d$, and the other, $r=o$, when $l=\pi d$, or the length of the rectangle of Fig. 3. πd is the theoretical maximum half length of belt for any distance between centers.

Having l assumed, as shown, and the reference line JS, fixed, R and r can vary from the limit

$$\left\{\begin{array}{l} R=o \\ r=\frac{l}{\pi} \end{array}\right. \text{ at } Z \text{ to } X\text{, where } R=r\text{, back to } Z \text{ again where} \left\{\begin{array}{l} R=\frac{l}{\pi} \\ r=o \end{array}\right.$$

including all possible values for this l.

Or if we have a given first pair of radii the reference line JS can be determined from them for their length of belt, and all other pairs of radii suitable for this length of belt will be at once shown. It is only necessary to take the difference of these radii and find the position of a vertical through the curve, such that the intercept VU equals this difference, Then measuring R downward from U, or r from V, the reference line JS is determined, and the length of belt can be shown at once by projecting the intersection S, up to L.

PRACTICAL DIFFICULTIES.

Now, while this diagram is remarkably complete in showing all of the intricate relations existing between the quantities of the problem, very clearly and perfectly, there are numerous objections to it, at once apparent from the standpoint of the draftsman who must deal with the problem.

Recalling the five requirements to which the desired solution must conform, which were determined upon in the beginning, page 42, we see that this diagram does not meet them fully. Investigating it closely in view of these requirements we have the following particulars in which each of them is, or is not, fully met by this diagram as now determined.

I. Any distance between centers may be assumed, but this would require plotting the irregular curve FXH, each time the method is used. Or else the scale each time would have to be

chosen to conform to the value of d on a permanent plotting, which would be inconvenient.

II. We cannot choose the first pair of radii in predetermined ratio conveniently on the diagram. This must be done aside arithmetically, or by a separate figure.

III. Having this first pair of radii, we can find the reference line and the length of belt, but only by a tentative operation, as has been explained. The difference of the radii (R—r), must be taken in the dividers and tried at various positions, till the location of a vertical is found such that this (R—r) laid off on it will just fit the two branches of the curve, as VU, Fig. 3.

IV. This condition is fully met. We have all the possible pairs of radii for the length l, found, easily attainable.

V. But this, the most important of the conditions, cannot be met at all except by many trials each time, and herein is the chief objection to the use of this figure. It is very necessary to be able to find succeeding pairs which shall be in definite ratio.

To overcome as many of these objections as possible, Fig. 3 is transformed, in the Reuleaux discussion, by replotting in such a way as to make it possible to obtain any number of successive pairs of radii, which shall be in predetermined ratio when once the reference line is fixed. But this transformed diagram embodies the two serious objections which have already been made to it, besides sacrificing the length of belt l, which, while not essential, it would be well to have retained: (1) It requires an irregular curve plotting, which necessitates either a permanent diagram, with the inconvenience of finding R and r in proportional parts, of d, or else a replotting each time. (2) The reference line for sucessive pairs of radii cannot be determined from the first or given pair, except tentatively, unless the case happens to be that of cones alike, with an odd number of steps, so the middle pair, with ratio 1 : 1 may be that taken for finding the reference line.

Considering closely these points of objection, and bearing in mind clearly just what is desired, it is seen that the one feature which is essentially unsatisfactory in both Fig. 3 and the Reuleaux transformation, is that both the radii of the pair R and r are measured in the same direction, and along the same line.

This makes the finding of an R and an r of definite ratio impossible in Fig. 3, and not as convenient as might be in the transformed figure.

If only R were measured *horizontally*, and r *vertically*, then it would be a simple matter to attain a desired pair of radii in definite ratio. This gives the final clue to the satisfactory form of diagram.

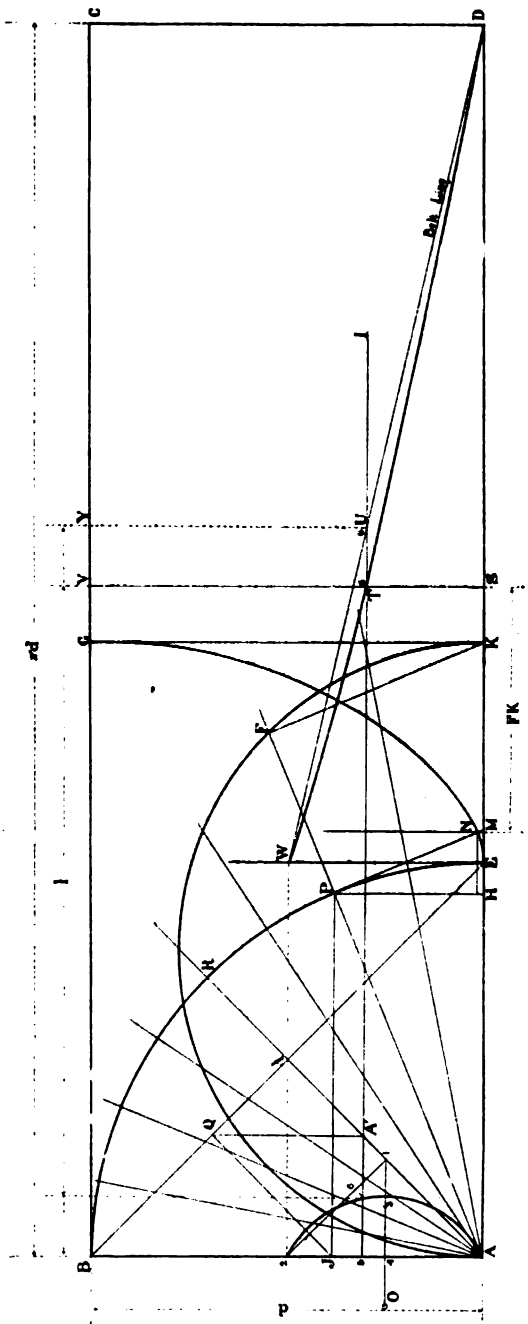

Fig. 6.

involute, ENG. Drop the perpendicular GK and on AK as diameter describe the semicircle AFC.

Now α, the angle of the belt, may be allowed to vary between its limits, o and 90°, and for each value of α taken, the new origin A′ and the value of the constant (R+r) can be seen at once. And then the constant length of belt for this new origin, and value of (R+r) can readily be had and plotted along the new horizontal reference line through A′ for the proper point on the belt line, as follows:

Consider any value of α, as $\angle$PAE. PH is now the value of (R+r), which must be constant. For PH=d sin α, and we know d sin α=(R+r) from Fig. 2.

Projecting P across to J, drawing JQ, or 45°-line, and dropping QA′, a vertical, down to the 45°-line AK, we have A′ for the new origin and A′L as the new reference line upon which it will be convenient to find the value for l, for this particular (R+r). PN, perpendicular to AF at P, gives the point N on the involute, through which drop the perpendicular NM. Then, as in the former case, AM=d(α sin α+cos α) which is the first term of equation (9) and FK is the other term, being

$$FK=d\frac{\pi}{2}\sin\alpha,$$

since $$AK=BG=d\ \frac{\pi}{2}\ ,$$

and $$FK=AK\ \sin\alpha;$$

$$\therefore\ FK=d\ \frac{\pi}{2}\ \sin\alpha.$$

Adding FK to AM we have point S, which gives T, a point on the belt line, from which the half length l is shown as BV.

Repeating this for succeeding values of α we have the curve WTD resulting, from which the length l can be obtained for any position of A′.

This curve is found to be of very gradual curvature, slightly convex downward. It is evidently limited by point W, vertically over E, and on a horizontal line through X, since the minimum value of l is d, when (R+r) vanishes, bringing A′ to X.

MODIFYING THE BELT LINE.

Examining the deviation of this curve from a straight line joining P and D, it was found after a series of trials that by the following slight addition to the diagram the straight line WD may replace the curve:

At the left of the figure draw the circle arc A62, the center of which is quickly located thus: Draw X2, horizontal; 21 a 45°-line; 14O a horizontal. Bisect 41 at 3, and mark O with O4 = 43. With O2 as radius draw the arc.

If, now, instead of beginning the measurement of each l from the vertical AB we measure from this circle arc, then the straight line WD gives the values of l and the curve may be abandoned. The length 6U will now be the value of l for the origin in position A′. And so for any position of A′ the value of l is seen on the horizontal through the origin A′, and is the part of this horizontal included between the arc A62 and the straight line WUD. Both this arc and the straight line are readily located and drawn, so that we now have for crossed belts a treatment exactly corresponding to that for the case of open belts, and equally as simple and complete.

THE FINAL DIAGRAM.

The essential portions of Fig. 6, constituting our useful working diagram, are shown in Fig. 7.

All lines can be easily and quickly drawn from memory with the simple drafting instruments. The rectangle is first constructed with AB equal the distance between centers, in any convenient scale, and the length BC=πd. Then with the T-square and 45°-triangle BE, AX, X2, 21, 1O, are at once drawn. Then point 3, the middle of 41 is marked, and O4 made equal 43, and the arc 26A struck. Then EW and XW are drawn, determining W, when WD may be drawn and the diagram is ready for use, just as the diagram for open belts is used, as already explained.

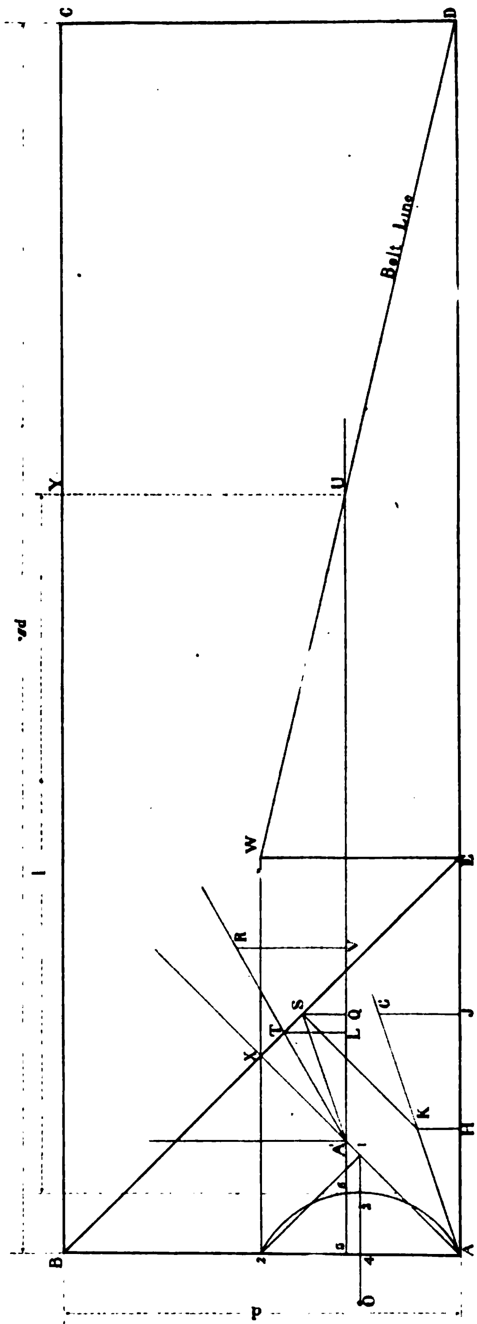

Fig. 7.

Rules for Proportioning the Steps of Cone Pulleys.

For ready reference, and for the use of those who may not care to follow the discussion presented in the preceding sections, the simple graphical method there deduced at length will now be taken as fully demonstrated and a brief rule given without proof, for each case of proportioning the steps of a pair of cone pulleys which may occur in practice.

REMARKS.

CONES ALIKE, OR UNLIKE.

Whenever the features of a design will admit it is always desirable to have the two pulleys of a pair just alike, so that the same pattern will serve for both. In such instances as a cone on a machine to run with another cone on the countershaft overhead, it is almost always possible to have the cones alike, and they should be made so. But, on the other hand, there are very many times when the nature of the machine on which the cones are to be used is such that it is impossible to make them alike.

SPEEDS IN GEOMETRIC SERIES.

In all cases, except when there is some very special reason why there should be a certain definite number of revolutions produced for the driven shaft, each time the belt is shifted from one pair of steps to the next the series of resulting speeds should form a *geometrical progression.*

That is, it is desirable that when the belt is shifted the number of revolutions of the driven shaft shall be a certain number of times (whole or fractional) the revolutions before; and that when it is shifted again to the next pair, the next number of revolutions will be the *same number of times* this speed.

This geometrical series is well established as always desirable, and should be attained whenever possible.

THE SIZE OE STEPS.

The relative size of the steps has no *apparent* relation, whatever, in general, to the proper series of speeds. If the steps of a cone are made with the same differences in diameter throughout the cones will, in general, be *wrong*. A pair of small three-step cones, with a large distance between centers, may have the same difference between the diameters of any two successive steps, but in no other case can this be true on both cones of the pair.

RELATIONS COMPLEX.

The relations of the problem are so intricate, made so by the fact that one belt must fit all pairs of steps with the same degree of tension, that it is impossible to figure correct diameters for cones, arithmetically; or to calculate them at all, except by rules and formulæ which are merely *approximately* correct.

FOLLOWING RULES, NON-APPROXIMATE.

The rules here given are the only ones which permit an entirely complete treatment of all cases, with resort to approximations. They are based upon a non-approximate demonstration and will be found to give full and satisfactory results under all conditions.

RULES TESTED.

To thoroughly test the reliability of these rules in practice the diameters for the cones of the foot lathe, shown in Fig. 10, were taken from the diagram carefully and a circular disc of heavy bristol card board was cut out accurately to the one one-hundreth of an inch for each step of each cone of the pair. Then the two discs of a pair were fastened to a large drafting board by means of a fine needle through the center of each, the needle points being accurately set a distance apart equal the distance between the centers of the cones on the machine. Then a length of very fine copper wire was placed around the two, representing the belt, and was drawn up to just enough tension to be straight between pulleys. Then each one of the pairs of discs, representing the pairs of steps, was substituted for the first, in turn, with the distance between centers exactly the same and the same fine wire representing the belt tried on each. The tension of the wire was tried in each case by "snapping" it between the pulleys. The result was that the tension of the wire was so nearly the same on each pair of steps that no appreciable difference could be detected whatever. And the length of the belt measured from the wire was the length shown by the diagram to within two one-hundredths of an inch.

For three years the rules here given have been used on designs of various characters with perfect satisfaction, both by the writer and others to whom they have been given.

OPEN OR CROSSED BELTS.

A design may call for a pair of cones with an "open" belt, which is usually the case; or a "crossed" or "twisted" belt may be required.

For open belts the diagram to be used is of one form and for crossed belts it is of another. But one is equally as simple as the other. And the one corresponds exactly to the other, so that if the use of one is understood the other may be used in just the same way. Both may be easily constructed with the ordinary drafting instruments, without the use of irregular curves or any special appliance. Being very simple both can be remembered easily after using a few times, so that they may be quickly drawn for any case in hand.

FORM OF DIAGRAMS.

The form of the diagram will be given for each of these two general conditions, and then the various cases of the problem will be taken up in turn and their treatment explained in reference to both diagrams, the two being so nearly alike and their use so similar.

NOTATION USED.

For convenience certain letters will be used to denote the quantities of the problem, and to avoid explaining them repeatedly they will be given here:

Let R be the radius of one step of the cone on the driving shaft;
and r the radius of the corresponding step on the driven shaft.
Let N be the number of revolutions the driving shaft makes;
and n the number which the driven shaft is to make when the belt is on this pair of steps.

Then let R_1, R_2, R_3, &c.,
r_1, r_2, r_3, r_4, &c.,
and n_1, n_2, n_3, n_4, &c., be corresponding values for the successive steps,
N being constant.

Let l be the half length of belt required,
and d the distance between centers, all in the same units and drawn to the same scale.

OPEN BELTS.

To design a pair of cones for an open belt draw first the rectangle ABCD, shown in Fig. 8, making AB=d the distance between centers of the shafts, using any scale most convenient. Make BC=πd, that is 3.1416 times the distance between centers. Then draw the "belt line" BD. Now draw the 45°-line FA, produced through C. Lay out the small square AC, shown, making the side of this square just $\frac{1}{10}$th of AB, the distance between centers. Then set

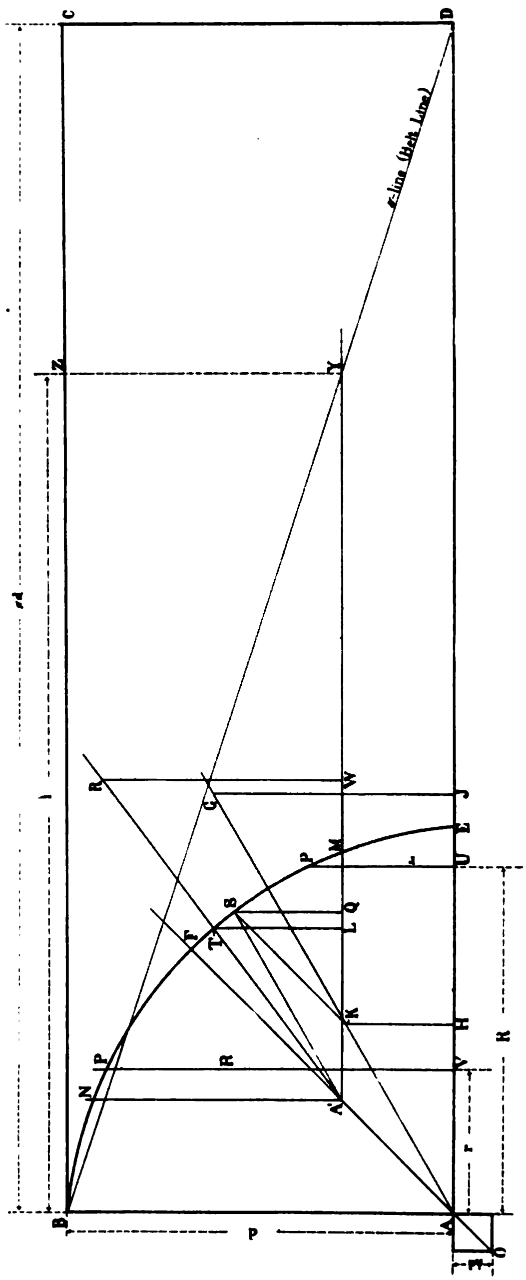

Fig. 8.

the center of the compass at C, and with the radius CB strike the arc BFE, and the diagram is ready to use.

CROSSED BELTS.

If dealing with a crossed belt draw the rectangle of Fig. 9, just

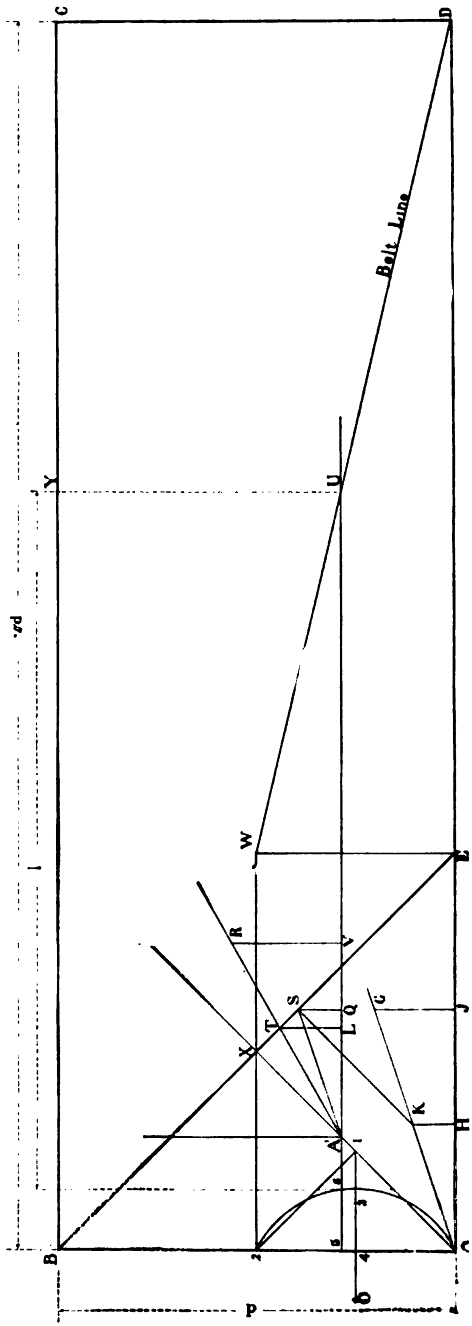

Fig. 9.

as for the case of open belts, making AB$=$d the distance between centers to any convenient scale, and BC$=\pi$d, that is 3,1416 times this distance between centers to the same scale. Draw a 45°-line, BE, from B, and a 45°-line, AX, from A, cutting BE at X. Draw a horizontal line through X and vertical from E, intersecting at W, and join W and D, thus obtaining the "belt line" WD.

Now from point 2, where WX produced cuts AB, draw the 45°-line 21. Through point 1 draw a horizontal line. Bisect 41 at 3, and with 43 mark off O from 4, making O4$=$43. Then with O as center and O2 as radius, draw the arc 26A, and the diagram is ready for use.

CASE I — Cones Unlike.

(a) SPEEDS ALL GIVEN.

When the speed of the driving shaft is given, and the speeds to be produced on the driven shaft are already determined by conditions of the design, and circumstances are such that cone pulleys of unequal size must be used, proceed as follows:

For illustration take the case of open belts. Construct the diagram as explained under the heading of "Open Belts," p. 65. (See Fig. 8.). From the nature of the design of the machine on which the cones are to be used determine what values are best to choose for the first pair of radii. If there are obstructions which the belt must pass, or the room for the cones is limited, this may require several trials. To do this draw a line from A, as AK, inclined so that any vertical to it as GJ, will be to its horizontal AJ, as $\frac{n_1}{N}$. That is, make AJ$=$N, and GJ$=n_1$ to any convenient scale. Then choose a size for R (or r may be chosen first) and lay it off from A to H with the scale used for AB. Then HK is the proper value for the other radius r, for this R, chosen. See if this meets the requirements of the machine. If not lay off another value for R and draw the vertical to the straight line for r. After a few trials a pair of radii will be found which will be of a suitable size for the requirements of the machine, and which will allow the belt to pass the obstructions, unless the design has in some way made this impossible.

This pair once found the other radii all depend on them. Suppose AH and HK to represent the pair R and r, found as just explained. Through K draw a 45°-line parallel to KA, and SA parallel to AK, finding A[1]. Now A[1] remains *fixed* for this particu-

lar pair of cones. Through A^1 draw a horizontal and produce it to its intersection with BD, when we have at once l, the half length of belt required for these cones, as shown.

Now having fixed A^1, the portion of the curve between N and M gives all the possible pairs of radii for steps which this length of belt will fit, horizontal measurements being the R's, and verticals the r's; and it only remains to pick out from all these possible pairs of radii those having the proper ratio to give the desired speeds.

This is quickly done thus: Take the next speed n^{11} and N the revolutions of the driving shaft. Lay off n^{11} from A^1 to W, and N up from W to R, using any convenient scale. Draw A^1R, and we have at once A^1Q for R^{11} and QS for r^{11} in the same scale as d and l, and the other radii. And so for each speed a pair of radii with which to draw the required pair of steps can be had at once from the curve BE.

If a case of crossed belts proceed in just the same way, using the straight line BE of Fig. 9 in place of the curve BE of Fig. 8 for the radii of the steps. The only difference is in finding the length of belt. Draw the horizontal through A^1 as before, and now the half length of belt, l, is the portion of this horizontal included between the intersection with the circle arc at point 6, and the point U on the "belt line" WD.

(b) SPEEDS TO BE IN GEOMETRICAL PROGRESSION.

Usually the speeds will have to be arranged to form a geometrical progression. Generally the end speeds will be given or determined upon independently, and the intermediate ones have to be adjusted to form this series. In this event if we are to have, say five, steps on the cones, we have known

$$N,\ n_1 \text{ and } n_5.$$

And we know that the five speeds will be:

$$n_1,\ n_1a,\ n_1a^2,\ n_1a^3,\ n_1a^4\ (=n_5),$$

if the series is to be a geometrical progression, 'a' being some multiplier. Then $a^4 = \frac{n_5}{n_1}$, and by use of a table of roots, 'a' can be found, and then each of the five speeds to the nearest convenient fraction of a revolution.

Now these values can be used as before, under (**a**), and the radii quickly found.

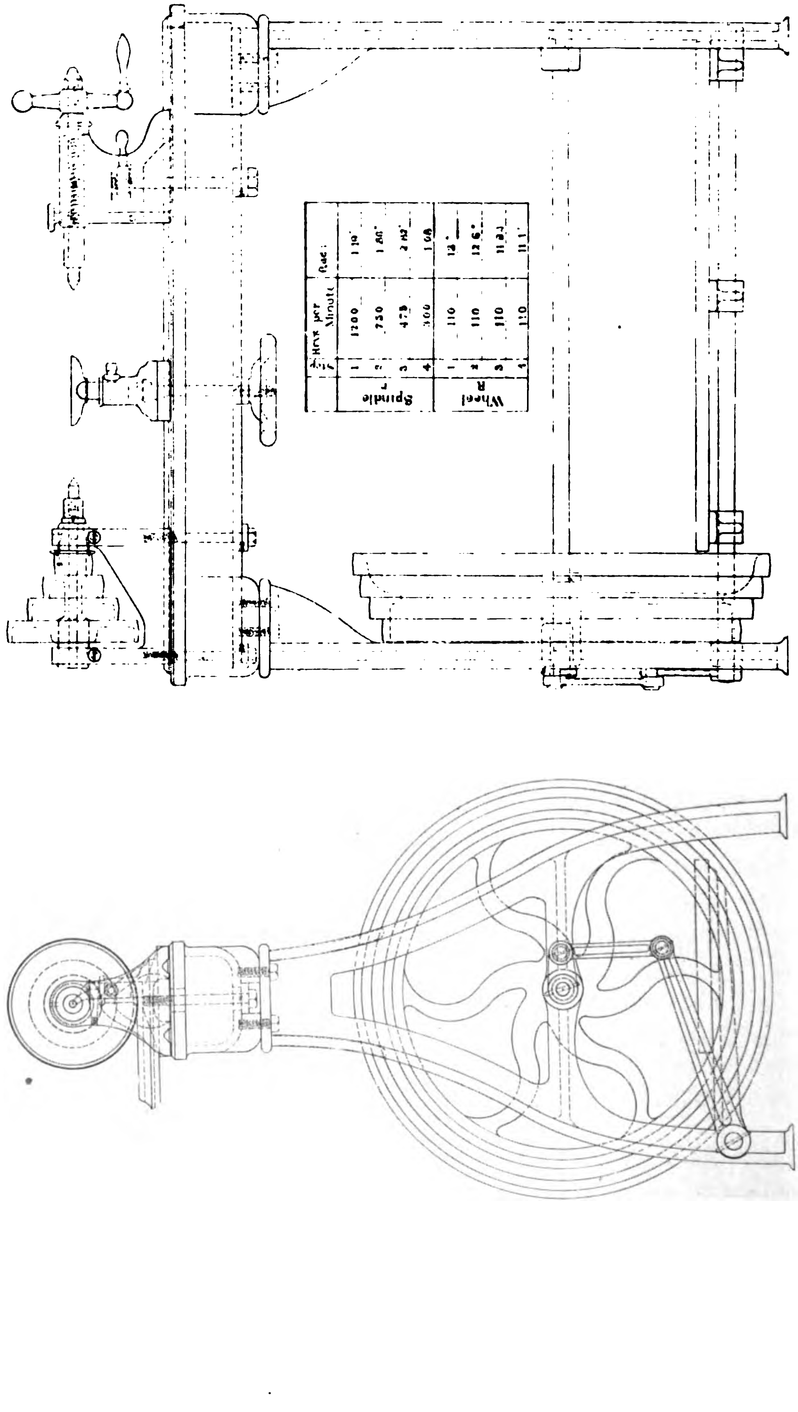

Fig. 10.

Fig. 10 shows two views of the assembly drawing of a foot lathe, the cones of which were designed just as explained. The fastest and slowest speeds were determined, and then the intermediate ones found as above.

With these values for n_1, n_2, n_3, n_4, and having determined independently what N should be the values of the radii were found by the means just explained in the preceding. It will be seen from the cut that it was necessary to proportion the cones in such a way that the belt would just miss the bed of the lathe. Several trials were made for the first pair of radii, just as under (a), in order to get the first pair of steps so that the belt would not conflict with the bed of the machine. Then the others came from the diagram readily, and the belt was found to just mill the bed when on any of the pairs of steps, the large driving wheel having been kept as small as possible.

CASE II—Cones Alike.

SPEEDS IN GEOMETRIC SERIES.

When the cones are to be alike,

$$R_1 = r_4$$

$$R_2 = r_3$$

$$R_3 = r_2$$

$$R_4 = r_1; \quad \text{(See Fig. 11.)}$$

and

$$\frac{R_1}{r_1} = \frac{n_1}{N} = \frac{r_4}{R_4} = \frac{N}{n_4},$$

$$\frac{R_2}{r_2} = \frac{n_2}{N} = \frac{r_3}{R_3} = \frac{N}{n_3},$$

$$\frac{R_3}{r_3} = \frac{n_3}{N} = \frac{r_2}{R_2} = \frac{N}{n_2},$$

$$\frac{R_4}{r_4} = \frac{n_4}{N} = \frac{r_1}{R_1} = \frac{N}{n_1};$$

From which

$$n_1 n_4 = N^2,$$

$$n_2 n_3 = N^2.$$

N is then the mean proportional between n_1 and n_4, the extreme speeds required for the driven pulley. If the pulleys are to have an odd number of steps the n at the middle pair would equal N. So now if the slowest and fastest speeds of the driven cone are

fixed by the conditions of the design, as is usually the case, N, the number of revolutions of the driving shaft or counter shaft, must be made the mean proportional between the extreme speeds. Simply multiply the extreme speeds together and extract the square root and the result is N.

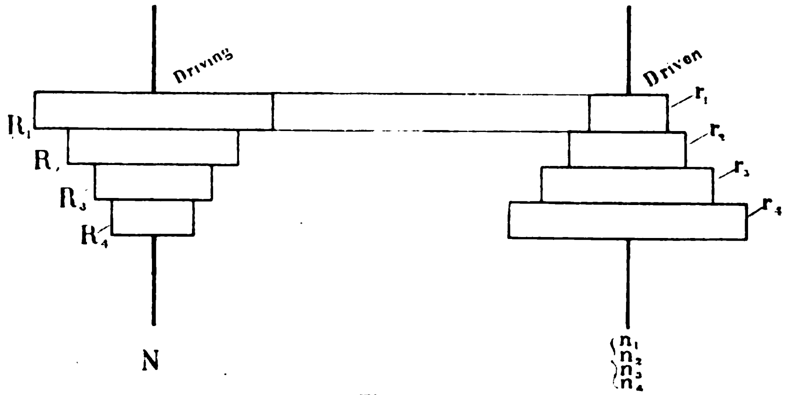

Fig. 11.

And besides this the speeds n_1, n_2, n_3, n_4 must form a geometrical series, so that

$$n_1 = n_1,$$

$$n_2 = n_1 a,$$

$$n_3 = n_1 a^2,$$

$$n_4 = n_1 a^3.$$

Then when n_1 and n_4 are the given extreme speeds

$$a^3 = \frac{n_1}{n_4},$$

from which 'a' can be found by use of a table of cubic roots, and then n_2 and n_3 figured.

Therefore, proceed as follows, there being four steps: Divide the fastest by the slowest speed and extract the cube root. If there are to be five steps extract the fourth root, if six steps fifth root, etc. With this result multipy the slowest speed. This will give the next highest. Multiply this now by the same multiplier and the result is the next speed in order. And so for all the intermediate speeds.

Now multiply the exteme speeds together and extract the square root, and this is the N, or number of revolutions the counter shaft should make.

Now construct the diagram for open or crossed belts. as the case may be. For illustration, take the diagram for open belts, Fig. 8. Lay off AJ = the number of revolutions n_1 and GJ=N, using any convenient scale. Then choose point K so that KH and AH will be of suitable size for the radii, r_1 and R_1, respectively.

Then
$$AH=R_1=r_4,$$
$$KH=r_1=R_4,$$

so two pairs of steps can be drawn. Through K draw KS a 45°-line, and through S draw SA′ parallel to KA, thus locating A′. Produce a horizontal through A′ to the belt line, and l, the half length of belt required, is shown at once in the same scale.

Now from A′ lay off $A'W=n_2$ and WR=N, using any scale. Join R to A′ and point T gives $TL=r_2$ and $A'L=R_2$.

Then
$$TL=r_2= \text{also } R_3$$
$$A'L=R_2= \text{also } r_3.$$

So now all the radii are determined and the steps may be drawn. If there were more steps the proceeding would be just the same. If there should be an odd number of steps one pair of steps would be equal, and their radii would be found given by point F. If the belt be crossed the operations are just the same, using the diagram of Fig. 9, where a straight line replaces the arc BE.

The only difference is in reading off the length of belt. In Fig. 9, for the position A′ the half length of belt is 6U, the position of a horizontal through A′, which is included between the arc and the straight belt line WD.

CASE III—Special Conditions.

SPECIAL RULES FOR SMALL THREE STEP CONES.

OPEN BELTS.

When the cones for an open belt are to have three steps each, with speeds in geometrical progression, and the distance between centers is to be great in proportion to the size of the cones then the steps may be figured by arithmetic, and this is the only case of cones for an open belt which can be. In general it is impossible to deal with cones for an open belt correctly in any way, except by the regular use of the diagram.

This one exception is due to the fact that when the distance between centers in Fig. 8 is great and the radii for the first pair of steps are small, we have a very large circle, BE, of which but a

small portion near point F is used. (Fig. 12 shows the conditions for this case.) This portion is so nearly a straight 45°-line. perpendicular to AF, that such a line may be used satisfactorily. This would mean an equal difference between the radii (or diameters) of the steps for a cone with three steps.

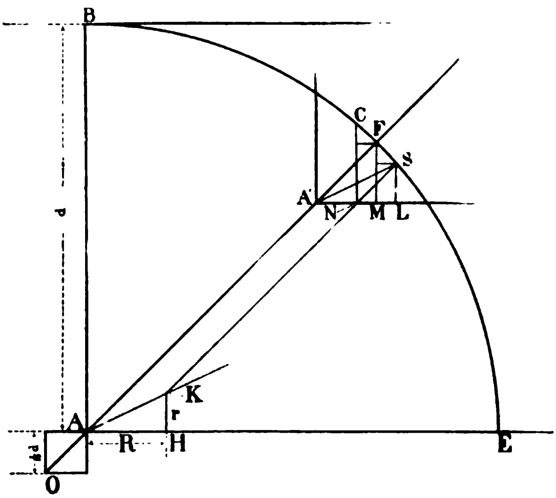

Fig. 12.

So it is only necessary to figure a pair of radii, R_1 and r_2, of a suitable size and so that $\frac{R_1}{r_1} = \frac{n_1}{N}$.

Then $R_1 =$ also r_3,

$r_1 =$ also R_3;

and $\frac{R_1 + R_3}{2} = R_2$, and $\frac{r_1 + r_2}{2} = r_2$.

CROSSED BELTS.

For a crossed belt any three step cone with speeds in regular progression may be figured thus, whether great or small in proportion to the distance between centers, for in the diagram for crossed belts a 45°-line replaces the circle arc. But only a three step cone can have a constant difference between the diameters of successive steps if the speeds are to be in geometrical progression.

Any pair of cones for crossed belts may be figured arithmetically, but for any case except that above explained for a pair of three

step cones it is not nearly so easy or satisfactory to do so as to use the diagram provided for the purpose.

LENGTH OF BELT.

According to these special rules for three step cones no attention is paid to the length of belt. It may be figured quickly for this particular case as follows:

Multiply the radius of the middle step by 3.1416 and add the distance between centers. This gives l the half length of belt.

CONCLUSION.

Any case of cone pulley designing will be found to fall directly under one or the other of the preceding cases and can be treated correctly by following the rules there given, even though the proofs are not known.

The rules can be used with more satisfaction, however, if the discussion which has been presented be followed first. With the full understanding of the intricacies of the problem and of the steps of the discussion leading up to the final conclusion, it will be much easier to appreciate the significance of the rules and to use them correctly. Hence it is urged that all who have to deal with the rules follow first through the full discussion, using, afterward, the rules for reference.

The Behavior of Kinoplasm and Nucleolus in the Division of the Pollen Mother Cells of Asclepias Cornuti.

BY WILLIAM C. STEVENS.

With Plate V.

In studying the development of the flower of Asclepias cornuti I was struck with certain pronounced characteristics of the kinoplasm and nucleolus in the pollen mother cell which seem to me to throw some light on the functions of those structures.

The functions of the nucleus as a whole have, to a certain extent, been made out with a fair degree of certainty. There is no doubt that the nucleus is the bearer of the inheritable qualities. The processes of fecundation and physiological experiment make this certain.[1] The nucleus also has important functions connected with processes which involve chemical changes, such as the formation of starch, the production of secretions[2] and possibly of proteids,[3] and the formation of plasma membrane and cell wall.[4] Regarding the specific functions of the different structures of the nucleus it seems clear that one purpose of the nuclear membrane is to enclose the nuclear sap so that it may not be lost to the nucleus by diffusion, and to keep the nuclear framework and nucleolus from being dissociated and drawn out through the cell by the movements of the protoplasm or during the translocation of the nucleus. By the action of reagents the nuclear wall may be made to shrink away from the surrounding cytoplasm, thus showing that the former may have a selective action in the passage of substances to and from the nucleus.[5]

It is possible, also, that the nuclear membrane is concerned in

1 O. und R. Hertwig, Die Zelle und die Gewebe.

2 Haberlandt, Ueber die Beziehungen zwischen Function und Lage des Zellkernes bei den Pflanzen pp 116-122.

3 Strasburger. Zellbildung und Zelltheilung, p. 373.

4 Tangl, E., Zur Lehre von der Kontinuitaet des Protoplasmas im Pflanzengewebe. (Sitzungsber. d. Kgl. Akad. d. Wiss. zu Wien. Mathem.—Naturwiss. Klasse. Bd. 90. Abt. I. 1885. p. 10.) Also Haberlandt. l. c., and Klebs. Beitraege zur Physiologie der Pflanzenzelle. (Untersuch. aus dem botan. Institut in Tuebingen. Bd. 2, p. 489.

5 Guignard, Nouvelles recherches sur le noyau cellulaire. (Ann. d. sc. nat. Botan. Ser. VI. T. 20. p. 310.)

the reception and transmission of stimuli. The fact that it is necessary for the nuclear membrane to be dissolved away during mitosis is evidence of its efficiency in demarking the nucleus as a distinct organ of the cell. It is, therefore, not necessary in accounting for the action of the nuclear membrane during nuclear division to anticipate for it any special function at this time, and it is highly probable that it undergoes only enough change to render it soluble, and that the substance of it is put to immediate use in the formation of the membranes of the daughter nuclei. It is possible, however, that the substance of it may take part in the formation of the spindle[1] and later enter into the construction of the membrane of the daughter nuclei.

The nuclear framework, from which the nuclear thread and the chromosomes successively arise, is unquestionably that part of the nucleus upon which the important duty of bearing the inheritable qualities specifically devolves. This is shown by the extreme exactitude with which the substance of the nuclear thread is divided in the formation and division of the chromosomes.

The character and behavior of the nucleolus do not make its function so evident as the functions of the nuclear thread and membrane appear. Although the nucleolus is a structure which is very rarely absent in the resting nucleus there is no evidence that it has a function to perform at such a time other than possibly the nutritive one of contributing to the formation of cell walls and starch grains.[2]

The conception is a fairly well grounded one that the nucleolus is reserve material to be used in the formation of new structures which arise during mitotic cell division. If this is a fact the preservation of the nucleolus within the nuclear membrane points to the nucleolus as a substance of great importance for the processes of mitosis, requiring the resting nucleus for its formation, and therefore not capable of being provided after nuclear division begins. The major part of the evidence taken from plants seems to show that the nucleolus is a stimulant and nourishment in the formation of the kinoplasm. Strasburger gives the following data for this conclusion: The enclosure of the nucleoli in the daughter nuclei after cell division appears to result in a diminution of the kinoplasm. The formation of the kinoplasmic spindle and the solution of the nucleolus occur simultaneously. After the solution of the nucleolus that portion of its substance which is not used in the

1 Flemming. Neue Beitraege zur Kentniss der Zelle. II Teil. (Arch. f. mikr. Anat. Bd. 37. p. 685.)
2 Strasburger, Ueber Kern und Zelltheilung im Pflanzenreiche, pp. 195-200.

formation of the spindle is still seen to be attached to it as nucleolar substance, and this occurs not only in the connecting fibres but also in the kinoplasmic radiations throughout the cytoplasm. By the use of the safranin-gentiana violett-orange staining method the violet color of the kinoplasm begins to decrease at the time when the nucleolar substance reappears in the daughter nuclei. In the formation of the plasma membrane about ascospores the nucleus remains in kinoplasmic connection with the membrane until the latter is fully developed, and this Strasburger considers to be evidence that there is an intimate relation between the substance of the nucleus and the kinoplasmic membrane, the nucleolus being the structure of the nucleus which is particularly concerned. Strasburger gives his conclusion from the above evidence in the following words: "Zwischen Kern und Kinoplasma besteht also, allem Anscheine nach, ein sehr nahes Verhältniss, und ich gründe auf dasselbe die Ansicht dass die Nucleolarsubstanz einen Reservestoff repräsentirt, aus dem das Kinoplasma nach Bedarf schöpft und durch dessen Aufnahme seine Thätigkeit erhöht wird."[1]

The fact that the kinoplasm is seen to arise during nuclear division in all parts of the cytoplasm[2] is good evidence that it is distributed equally throughout the cytoplasm when nuclear division is not taking place, although kinoplasm and trophoplasm are indistinguishable during the resting stage of the nucleus, excepting when successive nuclear divisions rapidly follow each other.[3]

Trophoplasm and kinoplasm probably remain distinct from each other, although it is not impossible that one may increase at the cost of the other. Strasburger has summed up the evidence afforded by plants for and against this view.[4] He calls attention to the fact that in dividing cells large quantities of kinoplasm appear which can no longer be made out in the resting cell, while the very opposite frequently occurs with the trophoplasm; a fact which seems to point to the mutual interchangeability of these two structures. However he calls attention to considerations which point the other way. The uniformity with which the kinoplasm in cell division is apportioned between the daughter cells; the formation by the kinoplasm of the plasma membrane which demarks the spores of Erysiphe and Peziza from the ascoplasm, without any apparent participation of the trophoplasm; the apparent exclusion

1 Strasburger, Cytologische Studien, pp. 224-225.
2 Osterhout, Ueber Entstehung der karyokinetischen Spindel bei Equisetum. (Cytologische Studien. pp. 6-7.) And Mottier, Beitraege zur Kentniss der Kerntheilung in den Pollenmutterzellen einiger Dikotylen und Monokotylen. (l. c. pp. 22-30.)
3 Strasburger, Ueber Cytoplasmastructuren, Kern und Zelltheilung. (l. c. p. 222.)
4 l. c. pp. 224-229.

of the trophoplasm by the spindle fibres from taking any part in the formation of the cell plate—these are considerations which seem to exclude the probability of a mutual interchange between kinoplasm and trophoplasm.

Besides the dynamic role which the kinoplasm seems to play in the separation of the chromosomes of the dividing nucleus it, perhaps, exclusively enters into the formation of the cell plate and the plasma membrane which, in free cell formation, demarks the new cell from the general cytoplasm. A beautiful demonstration of this last process has been worked out by Prof. Harper.[1] He finds that in the ascus of Erysiphe the daughter nuclei of the last division which is to produce the ascospores remain attached to their centrospheres by means of a projection of the nuclear wall. The chromatin framework of the nucleus extends up into the neck produced by the extension of the nuclear membrane and comes into intimate contact with the centrosphere. The radiations of the aster appear to be formed under the immediate influence of the nucleus and seem to grow outward from the centrosphere through the cytoplasm. Finally the kinoplasmic radiations of the aster bend backward toward the nucleus and increase in length. The radiations continue to grow, and at length they bend inward below the nucleus where they fuse together and form a complete membrane which demarks the nucleus together with a certain amount of cytoplasm, from the general cytoplasm of the ascus. When this process is complete the projection from the nucleus which was united with the centrosphere draws back and leaves the nucleus suspended in the cytoplasm. The plasma membrane of the spore is thus seen unmistakably to be formed by the kinoplasm. The fact that it remains in contact with the kinoplasmic radiations throughout this process is evidence that the nucleus has an important function to fulfill, either in inciting the growth of the membrane and in directing the course which it shall take, or in contributing to the material for the growth of the new membrane. It has already been observed that Strasburger conceives the union of the nucleus with the kinoplasm to be for the purpose of establishing direct communication between the nuclear substance and the growing membrane. The fact that the chromatin of the nucleus also comes in direct contact with the centrosphere is an indication that the nucleus is also exerting a formative influence on the growing membrane. Harper points out that the processes just described are evidence that the kinoplasm is a specific part of the cell and

1 Kerntheilung und freie Zellbildung im Ascus (Cytologische Studien, pp. 95-150).

not simply radially arranged cytoplasm, and that in this instance the kinoplasm has a nuclear origin.[1]

My own studies of the development of the pollinium of Asclepias cornuti have brought out some evidence in regard to the kinoplasm and nucleolus which will now be given. The pollen mother cells are formed very early in the development of the flower. Figure 1, plate V, is a young pollen mother cell from a cross-section of a pollinium from a bud which was about 2 mm. in diameter. The mother cells, which extend in a radial direction across the entire pollen sac, remain in a resting state until the bud has attained a diameter of about 4 mm. Then the mother cells divide twice, producing a radial row of four pollen grains. After the mother cells have been differentiated from the archesporial cells they pass through a long period of growth before they undergo their first division. Thus the mother cell grows in length from .085 mm. to .180 mm. and the nucleus of the mother cell grows in diameter from .011 to .017 mm. At the same time the nucleolus of the mother cell increases from .006 to .008 mm. in diameter, or nearly 2.5 times in bulk. While the nucleoli are relatively very large the amount of the chromatin in the nucleus is very little, as shown in Fig. 2, plate V, which represents a portion of a mother cell shortly before division. At this time the plasma membrane is the only means of separation between the mother cells and the tapetal cells. The latter form a layer from two to three cells thick and are likewise richly supplied with nucleolar material. At the time of the division of the pollen mother cells their relatively enormous nuclei become dissolved and no trace of them, as a rule, is to be seen, either in the spindle or in the cytoplasm, and so far as my preparations show this takes place before the solution of the nuclear membrane. The chromatin, which before the division of the nucleus was, for the most part, applied to the nuclear wall in the form of a loose net-work, breaks up into twelve small chromosomes, which are so small, indeed, that the manner of their division cannot be made out with certainty. It can be observed, however, that there is a reduced number of chromosomes formed in this division, for it is easily seen in a pole view of the nuclear plate of the archesporial cells that the number of chomosomes there is twenty-four. The small nuclear spindle converges to a sharp point at the two poles, and there is no trace of centrospheres, centrosomes, or of a multipolar origin of the spindle (Figs. 3, 4 and 5, plate V). My preparations leave me in doubt as to the origin of the spindle, but I am of the opinion that in this in-

[1] l. c. pp. 120-121.

stance it is wholly of nuclear origin, and that the nucleolus has contributed the substance for its formation. The abundance of the nucleolar material and its early solution suggest this, and no evidence of a contribution from the cytoplasm is to be seen. But judging from the amount of the nucleolar substance originally present it is probable that only a fraction of it is used up when the spindle is first formed. After the daughter nuclei have gone into the resting state the spindle still persists and increases in size, as shown by Fig. 6, plate V. The cell plate is formed, as usual, at the equator of the spindle, and with the spindle it continues to grow in diameter until it has traversed the cell (Fig. 7, plate V. While the spindle has been broadening it has been decreasing in length, as if it were being drawn in to form the cell plate. After the cell plate is complete the remnants of the spindle lose their distinct thread-like character and appear to merge gradually into the honey-comb structure of the cytoplasm (Fig. 8, plate V). When the spindle is first formed it takes on a violet color with the safranin-gentiana violett-orange stain, but after the daughter nuclei have gone into the resting stage the spindle fibres assume the pale brown color of the cytoplasm; so that both as to construction and reaction to stains the kinoplasm in this instance appears to merge gradually into the cytoplasm, or, in other words, the differences between kinoplasm and trophoplasm slowly fade away, as if these structures were morphologically the same thing. It will be observed that the daughter nuclei enter into the resting state and lose all connection with the spindle at an early stage in the formation of the cell plate, so that the growth of the plate proceeds without any kinoplasmic connection of the nucleus with the region of growth. This is the very opposite of the process of the formation of the plasma membrane about the spores of Erysiphe and Peziza as described by Harper, and stands out as evidence that the kinoplasmic connection of the nucleus with the growing membrane is not necessary in all cases. Reference has already been made to the opinion of Strasburger that the nucleolus contributes its substance to the formation of the plasma membrane of ascospores. We see in the pollen mother cells of Asclepias that the cell plate and the kinoplasmic spindle continue to grow after they have severed their connection with the daughter nuclei and the latter has entered into the resting state with their nucleoli already present. Accordingly if the nucleolar substance of the mother cell takes part in the continued growth of the cell plate and spindle a portion must have remained dissolved in the cytoplasm after the completion of the

daughter nuclei. I find it possible to get some direct evidence on this point. The nucleoli of the mother cells vary in size within quite narrow limits, and are fairly perfect spheres. It is, therefore, an easy matter to determine their volume with a fair degree of accuracy. I find the volume of an average nucleolus of the mother cells to be 265 cubic microns, while the sum of the volumes of the nucleoli of the daughter nuclei is about $\frac{1}{10}$ of this amount. Accordingly about $\frac{9}{10}$ of the substance of the nucleolus of the mother cell has been employed in some manner, and the most reasonable inference is that the new structures which have been formed during the division of the mother cell have been the recipients of this material. The new structures are, of course, the kinoplasmic spindle and the cell plate. It is imperative that these structures should be provided for before nuclear division sets in, since at that time the energies of the cell are probably greatly taxed by the processes of mitosis. The very large amount of nucleolar substance in the mother cells and the greatly diminished amount at the time of the completion of the daughter cells are facts which speak strongly for the nutritive function of the nucleolus.

The severance of the spindle fibres from the daughter nuclei before the completion of the cell plate is a fact which deserves special consideration. Haberlandt, after referring to observations by Treub and Strasburger which showed the attempt of nuclei during cell division to keep in close connection with the forming cell plate, comes to the following conclusion: "Da liegt nun die Annahme nahe, dass ausschliesslich in den Verbindungsfäden der Einfluss der beiden Tochterkerne fortgepflanzt werde und desshalb bloss innerhalb dieses Fadencomplexes zur Bildung der Zellplatte, resp. der Scheidewand führe."[1] The behavior of the pollen mother cells of Asclepias shows, however, that kinoplasmic connection with the nucleus is not always necessary to the formation of the cell plate, and that if the daughter nuclei exert an influence on the formation of the plate it is through the medium of the cytoplasm.

The persistence with which the nucleus remains attached to the kinoplasmic plasma membrane of forming ascospores may have the significance of establishing a direct highway for the transfer of nucleolar material to the growing membrane, but it seems to me that the formative influence of the nucleus is also transmitted in this way. The formation of a plasma membrane about an ascospore and the laying down of the cell plate in a dividing cell are in some respects quite different problems. In the former case there are no boundaries already existing which shall determine the form and

[1] l. c., pp. 110-112.

size of the developing membrane, and the nucleus as the bearer of the inheritable qualities must govern these characters. In the latter case, the direction of the cell plate having once been established, the problem is a comparatively simple one. Whether or not a kinoplasmic connection of the nucleus with the forming plasma membrane is necessary might depend in the individual cases upon the energy of the nuclear influence, the distance of the daughter nuclei from the growing membrane, and the sensitiveness of the cytoplasm in transmitting the nuclear influence. These are moments to which Haberlandt has called attention in considering the necessity of the proximity of the nucleus to those regions where growth is taking place most rapidly or is continuing the longest,[1] and their application in this instance is equally evident.

It seems fairly certain from the forgoing observations that the bulk of the nucleolar substance of the pollen mother cell remains in a state of solution after the daughter nuclei have gone into the resting state and is used, in part at least, in the formation of the cell plate. The proposition seems to me to be a reasonable and well-grounded one that the nine-tenths of the substance of the nucleolus of the mother cell which has not been used in the formation of the nucleoli of the daughter cells has been devoted to the building up of the new structuaes which arise during cell division. It may be that the kinoplasmic fibres sever their connection with the daughter nuclei so early in the formation of the cell plate because the nucleolar substance is to be obtained outside of the daughter nuclei.

If it is a fact that the nucleolus is merely reserve material which is destined to take part in the formation of the new structures of the dividing cell, then it must be regarded as reserve material of a very special potentiality. The long waiting of the mother cells and the gradual accumulation of nucleolar material already referred to are apparently a preparation for the critical time when the divisions of the mother cells shall follow each other in quick succession. At such a time the energies of the mother cell are probably taxed to their utmost so that the ordinary activities of the cell are in abeyance. The nucleolar substance is probably one of those which needs the influence of the nucleus for its formation, and at the time of nuclear division this influence could possibly not be given. This would then necessitate the previous preparation of the nucleolar substance as we have observed to be the case with the pollen mother cells of Asclepias.

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1 l. c., p. 102.

It does not appear to me that the occurrence in tissues of nucleoli in definite numbers is evidence against their nutritive function, as Schaffner has suggested.[1] On the contrary it might be anticipated that reserve material which is to be used in a definite time and space would be stored up in definite quantity and in the immediate neighborhood of or within the organ which is to govern its use.

[1] The Division of the Macrospore Nucleus, Bot. Gazette, vol. xxiii, No. 6, p. 438.

Physiography of Southeastern Kansas.

BY GEO. I. ADAMS.*

The National Geographical Society of Washington has published a volume of monographs entitled The Physiography of the United States. In this volume Maj. Powell of the U. S. G. S. has discussed physiographic processes and features and defined in a comprehensive way the physiographic regions and districts of our country. It is the purpose of this paper to discuss briefly the geologic structure of Kansas as relates to the regions with which it has a common history, and in particular to define the physiographic features of the southeastern part of the state.

PHYSIOGRAPHIC REGIONS TO WHICH KANSAS IS RELATED.

The regions to be considered in a discussion of the structural history of Kansas are shown in the accompanying map (page 89). The Ozark region embraces the Ozark plateau of southern Missouri and northern Arkansas and the Ozark ranges of eastern Indian Territory. The Ozark plateau extends just into the southeastern corner of Kansas. The Prairie Plains region extends to the north and west of this region. In Kansas it covers about the eastern fourth of the state. It is divided into a glaciated and a non-glaciated district, the division lines running approximately east and west just south of the Kansas and Missouri rivers. West of the Prairie Plains stretches the Great Plains Plateau terminating at the base of the mountains. That district of this region of which Kansas forms a part is known as the Arkansas Plateau. The Park Mountain region embraces the mountains of southern Wyoming, central Colorado and northern New Mexico. Between these ranges lie the mountain valleys commonly called Parks.

REGIONAL BOUNDARIES IN KANSAS.

The regions are defined and mapped in a broad way by Maj. Powell. Just what are considered the limits in each case is not

*Published with the consent of the Director of the University Geological Survey of Kansas.

told. Within the borders of our own state, however, they are manifestly well defined in nature and will admit of closer mapping. The regional boundaries are here also the boundaries of geological formations. In discussing the physical features of Missouri, Marbut* has made the line of separation between the Carboniferous and the Sub-Carboniferous the western margin of the Ozark region. In Kansas the line follows Spring River, thus giving the region but a very small extent within the state in the extreme southeast corner. The western limit of the Prairie Plains is apparently the escarpment along the eastern border of the Permian formation. This is a very natural division, and a traveler passing westward cannot fail to notice the sudden rise in the elevation and the change in surface features. In the southern portion of the state the transition is marked by the Flint Hills.

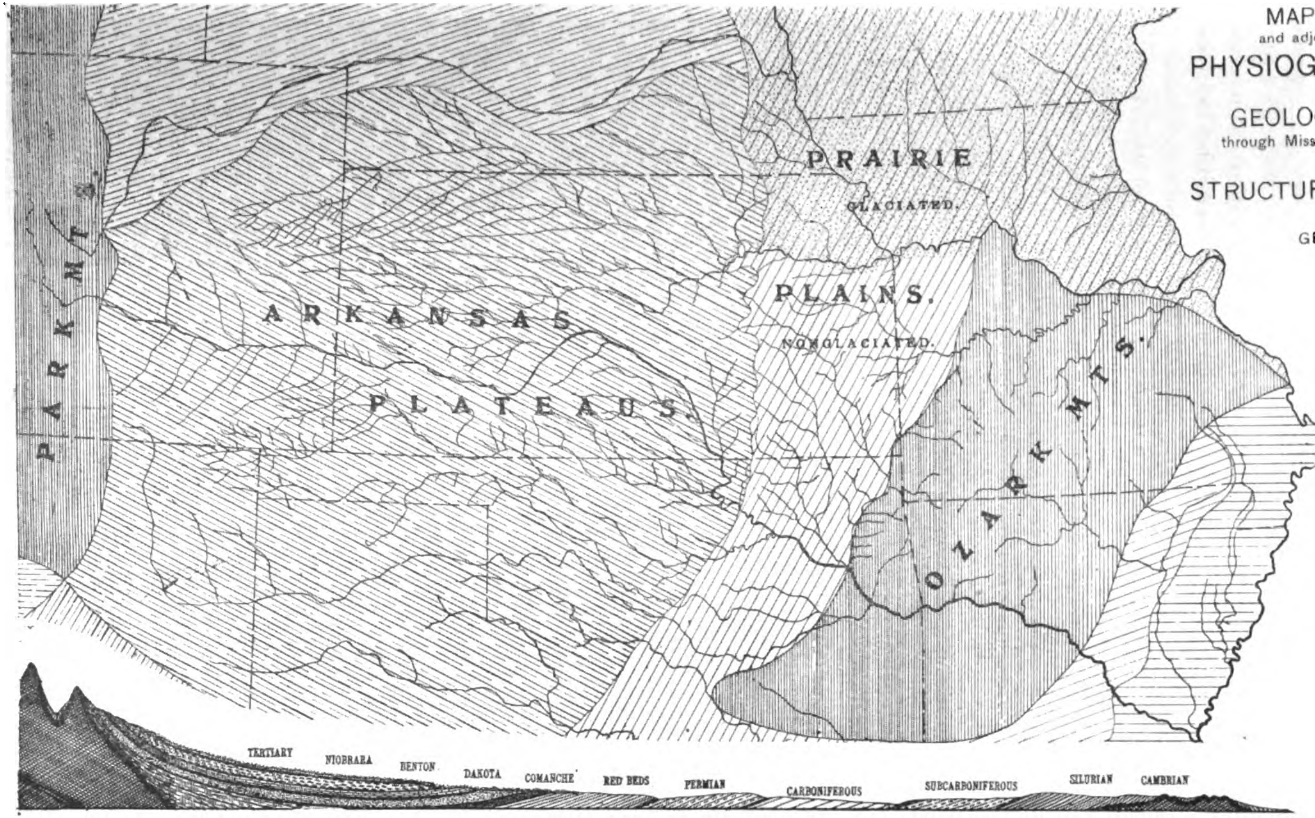
THE GEOLOGICAL STRUCTURE OF KANSAS.

The geological structure of Kansas is best understood by reference to a section extending in an east and west direction from the dome of the Ozark region to the Park mountains (see map p. 89). From this section it will be seen that within the Ozark Uplift is a core of archean rocks which are exposed within limited areas in southeastern Missouri. Around this core is an area of the older Palæozoic formations. Passing westward we find the beveled edges of the later Palæozoic formations. Along the line of the section they are represented as follows. First: undifferentiated Cambrian and Silurian and the Sub-Carboniferous. Within the Prairie Plains region lies the Carboniferous. The Arkansas Plateau region embraces the Permian, the Red Beds which are referred by some to the Permian and by others to the Triassic, the Comanche, Dakota, Benton and Niobrara, while resting unconformably upon these is an irregular deposit of the Tertiary. At the base of the Park mountains are the upturned edges of the Cretaceous and older rocks, while within the region the formations are much disturbed and the ranges contain eruptive and Archean elements. Kansas may be said to be an area of slight disturbance lying between two mountainous regions, whose complex histories have produced simple oscillations over the regions of the Prairie Plains and the Arkansas Plateaus.

We cannot reconstruct with much certainty the original areas of these various formations, but they once extended much further to the east, and to produce their present surface and beveled out-

*Physiographic Features of Missouri. Mo. Geol. Sur., Vol. 10.

MAP OF KANSAS
and adjoining states showing
PHYSIOGRAPHIC AREAS,
with a
GEOLOGICAL SECTION
through Missouri, Kansas and Colorado
showing the
STRUCTURE OF THE AREAS,
BY
GEO. I. ADAMS.



crops, erosion has been at work at varying intervals and for long periods. The westward dips and the succession from older to newer formations along this section argues in favor of the hypothesis that the shore line during Palæozoic times was to the east. This hypothesis is still further strengthened by the fact that the deposits themselves, show marginal conditions in their eastern outcrops, while the records of deep wells show deep sea conditions to have been more prevalent to the west as is indicated by the thinning of shale beds and the thickening of the limestone systems.

That this shore line made many oscillations and migrations is evidenced from the alternation of oceanic and litoral deposits and the deposits of coal in the upper part and near the westernmost exposures of the Carboniferous. At the close of the Palaeozoic era the land area must have advanced much further westward, since the deposits of salt and gypsum in the upper portion of the Permian indicate the absence of open seas. During the whole of the Cretaceous period deep sea conditions prevailed over most of the state since the deposits are now present in the western two-thirds of it after a considerable erosion. At its close the raising of the mountains to the west caused the final retreat of the sea. The only remaining deposits, the Tertiary and limited Quarternary areas being of fresh water origin.

ORIGIN OF PRESENT DRAINAGE.

Until the close of the Cretaceous the drainage of Kansas or such portions of it as were land areas, was to the west into the Cretaceous sea, since the deposits indicate a land mass to the eastward. The raising of the mountains to the west produced a drainage slope to the east over the newly exposed Cretaceous formations, which subjected them to a considerable erosion before the Tertiary of Kansas was laid down. Just what oscillations have occurred since then are not so easily determined. If, however, the Tertiary deposits were lacustrine to any extent, it would seem probable that there existed during that period a broad basin extending over the western part of the state far to the north and south, into which the drainage from the west flowed. If the sediments which produced the Tertiary were simply spread out on a flood plain, similar conditions probably existed. It appears therefore that not until near the close of the Tertiary times were the Park mountains sufficiently elevated to induce a drainage from that region across the Arkansas Plateaus to the Mississippi. We may accordingly look upon the present physiography of Kansas as being of the latest period.

DRAINAGE OF THE PRAIRIE PLAINS.

The drainage of the Prairie Plains is due primarily to the eastward slope of the surface. By reference to the map it will be seen that with the exception of the Kansas river all the streams rise within the area. A secondary feature is the dip and strike of the rocks. In general the dip is to the west and the streams flow at right angles to the strike, but slight deformations of the strata have caused a deflection of some of the streams to the south. There is an anticlinal ridge which has determined the divide between the Neosho and Osage river systems. In Missouri this divide continues into the Ozark region to which the anticlinal is no doubt structurally related. Near the eastern border of the state this divide is spoken of by the residents there as the Ozark Ridge, and they will tell you that it can be traced to the Ozark mountains, but many of them mistake the escarpments which cross the ridge for the ridge itself. Along the southern border of the state the dip is to the southwest and the streams here become more directly tributary to the Arkansas which finds its way through the Ozark region in a synclinal trough.*

Spring river, which crosses the southeast corner of the state, flows along the line of contact between the Sub-Carboniferous and Carboniferous formations and has literally slid down the extreme border of the Ozark dome, eroding the shales of the Carboniferous and accommodating itself to the uneven surface of the Sub-Carboniferous. In the Territory the Neosho, to which Spring river is tributary, occupies a similar position and is deflected to the west considerably before it reaches the Arkansas.

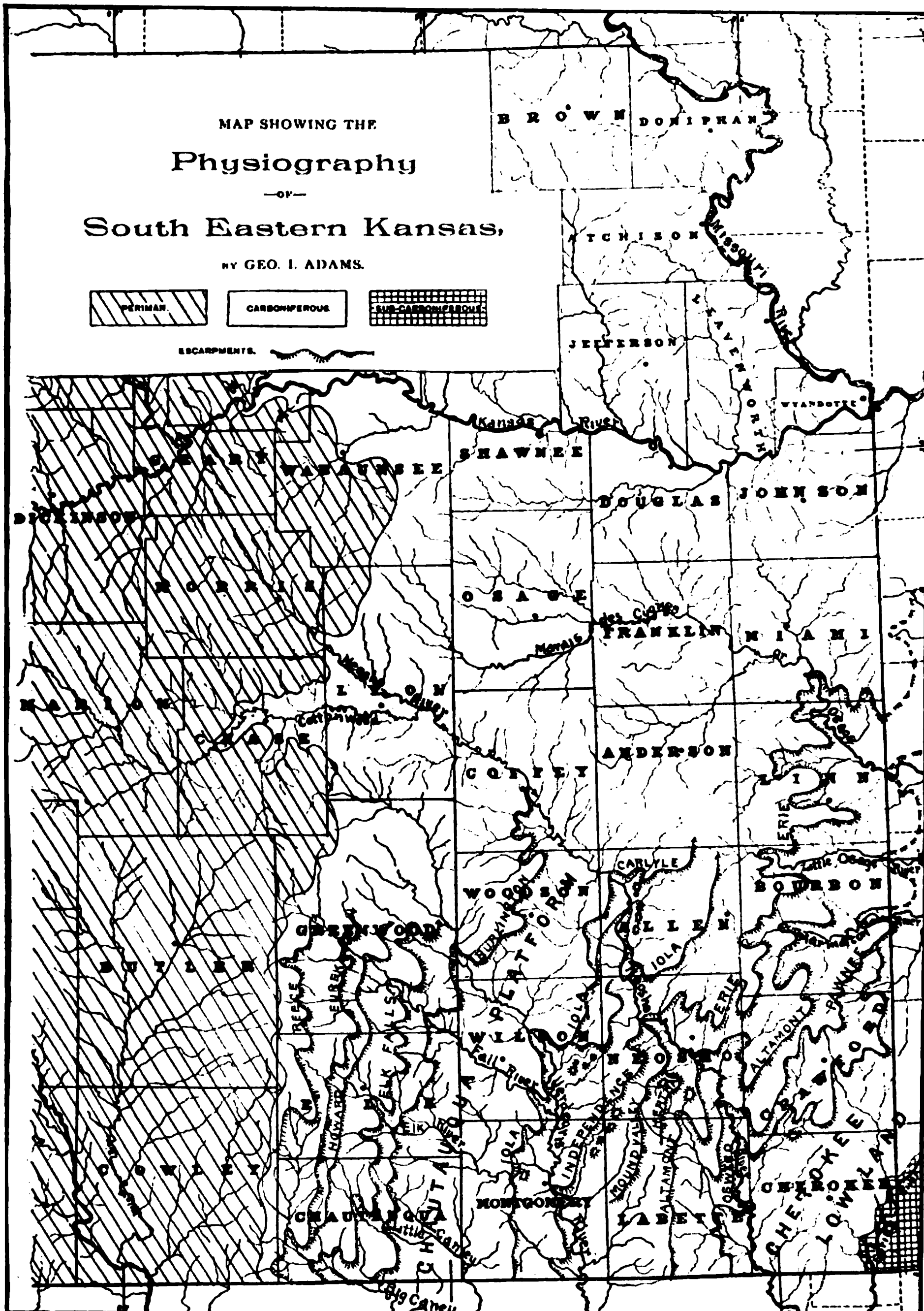
RESULTANT TOPOGRAPHY.

The formations over which the streams flow are beds of limestone alternating with beds of sandstone and shale. The unequal yielding of these materials to erosive agencies has produced in general a terraced surface, the limestones protecting the escarpments while the shales and sandstones below have been carried away by the streams.†

The inclination of the strata has produced a gradual slope (back slope) from the top of one escarpment to the base of the next higher. Not infrequently a stream cuts off a portion of an escarpment producing a mound or ridge, and the ridge in turn is broken up into a row of mounds. In case the more resistant strata are

*Geo. H. Ashley, Geology of Paleozoic Region of Arkansas, Proc. Am. Phil. Soc., May, 1897.

†*Vide* Haworth, University Geol. Sur. of Kansas, Vol. I, Chap. 10.


MAP SHOWING THE
Physiography
—OF—
South Eastern Kansas,
BY GEO. I. ADAMS.
PERMIAN.
CARBONIFEROUS
SUB-CARBONIFEROUS
ESCARPMENTS.
BROWN
DONIPHAN
ATCHISON
JEFFERSON
LEAVENWORTH
WYANDOTTE
Missouri River
Kansas River
SHAWNEE
WABAUNSEE
DICKINSON
DOUGLAS
JOHNSON
MORRIS
OSAGE
FRANKLIN
MIAMI
LYON
MARION
CHASE
COFFEY
ANDERSON
LINN
GREENWOOD
WOODSON
ALLEN
BOURBON
BUTLER
WILSON
NEOSHO
CRAWFORD
ELK
COWLEY
CHAUTAUQUA
MONTGOMERY
LABETTE
CHEROKEE
PLATFORM
IOLA
ERIE
ALTAMONT
PAWNEE
INDEPENDENCE
OSWEGO
MOUND VALLEY
CARLYLE
ELK FALLS
EUREKA
HOWARD
Fall River
Elk River
Big Caney
Little Caney
Cottonwood
Neosho River
Marais des Cygnes
Little Osage River
LOWLAND

anywhere discontinued or lose their importance the escarpment fades out, and the softer beds add their thickness to the escarpment geologically next higher. Likewise if the softer strata give place to more resistant ones, e. g. a limestone system appears in a position where in other cases there is a shale bed, a new escarpment is developed. If a shale bed gradually thins out the adjacent limestones are merged into the same escarpment, and, on the contrary, if two closely associated systems are separated at any place by the thickening of the intervening shales, their lines of outcrop diverge and two escarpments are produced.

A stream flowing upon a back slope will gradually slide down upon the inclined strata until it reached the base of the next escarpment, cuts through the underlying formation, or reaches base level. In the latter case it would widen its valley, producing a plane independent of the dips of the strata. Along a single stream this area would be called a bottom land. When produced over a large area by a stream and its tributaries, or by several streams it is called a low land.

The Prairie Plains region in Kansas is coextensive with the Carboniferous formation. Were a section made from Galena to Grenola it would pass across the entire formation as here exposed. At its eastern limit we have the break in geological time marked by unconformity, at its western limit the transition to the Permian is marked by the fossils only. The dip of the strata and the alternation of the easily eroded shales and sandstones with the more permanent limestones, is well exhibited by such a section drawn to scale. The lines of outcrop of the limestone systems have been traced with considerable detail and are quite identical with the escarpments which are shown in the accompanying map (page 92). It will be observed that they trend in a northeast and southwest direction with many sinuosities where they cross the streams and divides.

DESCRIPTION OF PHYSIOGRAPHIC FEATURES.

Proceeding now to a description of the physiographic features and beginning with the southeastern corner of the state we find a small area of Sub-Carboniferous which belongs to the Ozark region and which forms our starting point (see map page 92). To the west of this lie

THE CHEROKEE LOW LANDS.

The base of the Carboniferous consists of a bed of shales and

sandstones about 450 feet thick, known as the Cherokee shales* and sandstones. They are exposed over a belt of country about twenty-five miles wide, lying between Spring river and Oswego, and extending across the corner of the state far into Missouri and the Indian Territory. The surface is gently undulating, the monotony of lowland topography being occasionally broken by ridges and mounds which owe their existence to heavy sandstone. Such a mound is the one west of Baxter Springs near the Territorial line. The country around Columbus exhibits a number of sandstone ridges, the city being located upon the divide between Spring river and the Neosho. Within this area are situated Pittsburg, Cherokee, Columbus, Wier City, Mineral, Sherman and Chetopa. Over this area there are no limestones of stratigraphic importance, those which exist being usually associated with coal seams as "cap rocks." The western border of the Cherokee lowlands is

THE OSWEGO ESCARPMENT.

At Oswego we meet with the first important limestones, known from their exposure at that place as the Oswego formation.†

They cap the heavy shale beds and produce an escarpment which along the river bluff is 120 feet high. To the south the escarpment continues west of the Neosho river passing into the Territory; to the northeast it passes Sherman, and produces the hills around Monmouth and to the northwest of Mineral City. Just north of Cherokee it bends to the north, crosses Cow Creek near Girard and runs a little north of east almost to the state line, being very prominent at Mulberry, from which place it bends to the north, passing in a sinuous line west of Arcadia, Bunker Hill being a part of it. From there to Fort Scott it is very irregular and has many outlying hills. East of Fort Scott it reaches the state line, but bends back up the Marmaton and after crossing the river, finally passes into Missouri. It is in this escarpment that coal is obtained by quarrying the limestone above it. The Oswego formation extends but a short distance to the west on the surface. The next formation met with is the Pawnee, producing

THE PAWNEE ESCARPMENT.

The Pawnee limestone‡ formation produces but a slight influence upon the topography. To the north it becomes more important and is seen in the escarpment crossing the divide between Labette

*Haworth and Kirk, Univ. Quart., Vol. II, p. 105.
†Haworth and Kirk, Kansas Univ. Quart., Vol. II, p. 105.
‡Haworth and Kirk, Kansas Univ. Geol. Surv., Vol. I, Chap. 2.

creek and the Neosho near Laneville. East of the Neosho it passes McCune to the south, and runs up Lightning Creek crossing it near its head, from which place it follows a sinuous line north of Girard, west of Englevale and east of Pawnee station to Godfrey, where it blends with the Oswego escarpment.

THE ALTAMONT ESCARPMENT.

The next limestone formation is the Altamont.* It is found at the top of the escarpment which passing east of Altamont and extending to the south, runs just east of Edna and into the Territory, becoming very prominent there. Tracing the systems to the north they are found to pass from Altamont north to Parsons, at which place they produce no escarpment. Along the west bluff of the Neosho they are again important, but the escarpment fades out around St. Paul in the Neosho bottoms. To the northeast, past Brazilton, Farlington and Hiattsville, there is a prominent escarpment but it is largely due to the thick sandstone beds which produce the flagging stone quarried at many places. In the valley of the Marmaton the escarpment disappears but is again found further north.

THE ERIE ESCARPMENT.

This escarpment is most prominent along the Marmaton river near Uniontown, from which place it runs to the northeast, passing out of the state east of La Cygne, according to Mr. Bennett who is familiar with that region. Following it to the south it passes east of Savonsburg nearly to Walnut, thence westward north of Erie, where it crosses the Neosho river. It is produced by three limestone systems called collectively the Erie formation.†

They are quite closely associated along the course already described and the escarpment is one of the most prominent in the southeastern portion of the state. South of the Neosho, however, the systems separate, due to the thickening of the intervening shale beds, and the lines of their outcrops diverge.

THE HERTHA ESCARPMENT.

The lower member trends to the east around the head of a creek to Hertha. An outlying area is found at South Mound. From Hertha the escarpment runs westward crossing Labette creek south of Galesburg and follows its west bluff for a considerable distance. It is prominent at a few places on Little Labette Creek and finally terminates in the mounds west of Altamont.

*Adams, Kansas University Survey, Vol. I, Chap. I.
†Haworth and Kirk, Kansas Univ. Quart., Vol. II, p. 108.

THE MOUND VALLEY ESCARPMENT.

The second member of the Erie formation, the Mound Valley,* has very prominent exposures all the way from the Neosho river to Galesburg and for some distance south when it suddenly fails to produce an escarpment for a considerable way, especially near Little Labette Creek. At Mound Valley, however, it is very prominent continuing so to the southwest nearly to the Verdegris river at Liberty. Here, however, the limestone has disappeared and the escarpment is produced by sandstones, which are eroded further south by the tributaries of the Verdegris.

THE INDEPENDENCE ESCARPMENT.

The upper system of the Erie formation, the Independence,† is found on the high land east of Urbana and south of that place, producing an escarpment east of Thayer, which runs in a southwest direction to the Verdegris just below Independence. Crossing to the west side of the river, it produces a high bluff all the way to Coffeyville and after a slight digression to the west at Onion creek, passes into the Indian Territory. Lying to the east of this escarpment from south of Thayer to the Verdegris, is a chain of hills and mounds, including the Bender mounds and those around Cherryvale, which form a very striking feature of the country. They are simply the remains of outlying areas separated from the escarpment by erosion.

EARLTON ESCARPMENT.

The next succeeding formation is the Iola limestone formation. Below the Iola system proper lie the Earlton limestone systems.‡ From Elk river to east of Benedict they are closely associated with the Iola, but west of Chanute and northwest of Earlton, from which place the systems take their names, they produce a separate escarpment, due to the thickening of intervening shales. Between Altoona and Earlton there are a number of mounds which have recently been protected by the limestone which produces this escarpment. The escarpment fades out in the Neosho Valley, and southwest of Vilas blends with the next succeeding, which is the

IOLA ESCARPMENT.

The escarpment produced by the Iola limestone‖ is most prominent west of the Verdegris river from Table Mound northwest of Independence to Benedict. South towards the state line it loses

*Adams. Univ. Geol. Sur., Vol. I, Chap. 1.
†Adams. ibid.
‡Name here proposed.
‖Haworth and Kirk. Kansas Univ. Quart., Vol. II, p. 109.

much of its importance, due to the preponderance of sandstone. It passes into the Indian Territory just east of Tyro. Walker Mound and Table Mound are outlying areas of it. The prominence of the escarpment west of the Verdegris from Independence, past Neodesha, Altoona and Guilford to Benedict, is largely due to the position of the river valley, which runs parallel to it, and to the great thickness of the limestone. At Benedict it crosses the river and, bending somewhat to the south, takes a northerly course, passing west of Vilas, to Owl Creek west of Humboldt, where the next succeeding escarpment blends with it. The Iola limestone descends to the bottom land and is not traceable far after crossing Owl Creek on the west side of the Neosho river. It is exposed at Iola along Elm creekand Rock Creek, as well as at several places on the east bank of the Neosho between Iola and Humboldt; at the latter place it forms the heavy ledge which is so prominent at the river bridge. Further south it recedes from the river, producing an escarpment which trends to the southeast, then curves to the northeast, running nearly parallel to Big Creek, but considerably west of it and passes just east of Moran, becoming less distinct.

THE CARLYLE ESCARPMENT.

This escarpment is produced by the Carlyle limestone,* which is exposed near Carlyle on both sides of Deer Creek. From that place it runs to the east and then to the north, being prominent at Garnett and east of there along the Pottawatomie river. On the north side of Deer Creek it trends west from Carlyle to the Neosho river, crossing it below Neosho Falls. On the west side of the Neosho it follows the river bluff for a short distance, then runs to the south, passing about half way between Iola and Piqua. At Owl Creek it blends with the Iola escarpment as already stated, although the system is traceable somewhat further to the south. North of Iola and between Iola and Humboldt east of the river, there are a number of hills which are outlying portions of this escarpment.

THE CHAUTAUQUA PLATFORM AND THE CHAUTAUQUA SANDSTONE HILLS.

The back slopes of the escarpments thus far described possess no features which merit special description. The shale beds which produce them contain usually but little sandstone, except along

*This limestone was named the Carlyle limestone by Haworth and Kirk, Kansas Univ. Quart., Vol. II, p. 110, and the first succeeding one above the Garnett or Burlington. It has since been learned, as will soon be published by Haworth, that the so-called Carlyle limestone is the lower member of the Garnett.

the southern border of the state, and the limestones succeed each other at short intervals, so that the platforms are not very wide and their surfaces are generally undulating. West of the Iola escarpment and the Carlyle which blends with the former, lies an area which is more diversified, due to the manner in which erosion has acted upon the heavy beds of sandstone which are present as the equivalent of the Le Roy Shales* further north. North of the Neosho river sandstones are represented but sparingly in the Le Roy shales, but south of the river they gradually displace the shales, until in Chautauqua county they are everywhere predominant. From their exposure here they are named the Chautauqua sandstones.† At Yates Center they become conspicuous, producing the hills upon which the town is built. From here the area broadens to the south, its eastern border passing west of Buffalo, Fredonia and Tyro, while its western border runs approximately from Yates Center to Toronto, Fall River, Elk Falls, Sedan and Elgin. The area will here be described under the geographical name of the Chautauqua Sandstone Hills. These hills are as characteristic a feature of the southwest part of the state as are the Flint Hills,‡ and I here propose the name as one best applicable since it is already employed somewhat in common usage for a portion of the area. The surface is intersected by many small streams, which have deep valleys. The Verdegris, Fall and Elk rivers cross it, occupying narrow, deep channels, which are down to base level except along the western portion. The valleys of these rivers are narrow and walled in by bluffs, which show heavy sandstones as their protecting element. The low hills which are the prominent feature of the area are usually covered with a growth of jack-oaks. The sandy soil is seemingly adapted to their growth, for where the limestone areas are approached the oak timber begins to disappear. There are some small areas outside of the Chautauqua platform which have a similar growth of timber, as west of Thayer and south of Independence along the west bluff of the Verdegris, where the Thayer shales,|| which lie between the Independence and Iola limestones, carry a great deal of sandstone. Although the Chautauqua sandstone hills are nowhere very high, the difference in elevation over the entire surface being nowhere greater than 250 feet, yet they make traveling difficult because of the rocks which wear to the surface on the slopes, and the sand

*Haworth and Kirk, Kansas Univ. Quart., Vol. II. p. 110.
†Name here proposed.
‡Adams, Kansas Univ. Sur. Vol. I, Chap. 1.
||Haworth, Kansas Univ. Geol. Sur., Vol. I, p. 157, see foot note.

which accumulates in the wagon roads from the disintegration of the sandstones.

THE BURLINGTON ESCARPMENT.

The limestones exposed in Burlington* and just south of the town, form the protecting element in the next escarpment. This escarpment, known as the Burlington, is prominent west of Le Roy Junction and along Turkey Creek. It runs to the southwest, passing two miles west of Vernon, and then around the head of Owl Creek. Tne limestone is present three miles west of Yates Center, but the heavy sandstones which produce the hills at Yates Center mask the escarpment, as indeed they do in most places from there to the southern border of the state. This limestone formation is the upper limit of the Le Roy shales and Chautauqua sandstones, but the general character of the Chautauqua hills area persists to the next succeeding escarpment. The line of outcrop of the limestone is from Yates Center to Toronto, thence west of Coffeyville to Fall River, Longton, Sedan, Chautauqua Springs and Elgin. In places the limestone being underlaid by shales or softer sandstone, persists as a prominent element in the surface features, but where the sandstones are in heavy ledges, it loses its relative importance. From Sedan to Elgin it is easily traced.

ELK FALLS ESCARPMENT.

The next escarpment is produced by two heavy limestone formations which are usually separated by a sandstone formation, which weathering slowly, brings all three ledges into practically the same slope. These formations and subsequent ones are not here named, since it is not necessary to a discussion of the present subject and a strict correlation is not now possible.† The escarpment is prominent west of Elk Falls. The two limestone formations produce the two heavy ledges seen along the railroad from Elk Falls to Moline. From Elk Falls southward the escarpment passes with many deep sinuosities around the head of Salt Creek, North Cana, Middle Cana and Cedar Creek, and leaves the state west of Elgin after having digressed up Big Cana nearly to Hewins. It is seen very prominent at Rogers, about five miles west of Sedan. Northward from Elk Falls it passes up Elk River nearly to Howard, then descends the river again, is found west of Hutchins Creek, at Cave Spring on the head of Indian Creek, at Greenwood on Salt Creek and west of Fall River to the vicinity of Twin Falls. Thence it

*Haworth and Kirk. Kansas Univ. Quart., Vol. II, p. 110.

†Probably these systems have already been given names where exposed along the Neosho and Kansas rivers in Kansas Univ. Sur., Vol. I.

makes a broad bow to the east and so reaches Walnut Creek, south of Neal. From there it trends to the northeast but the character of the limestone and included sandstone formation is changing somewhat, so that it is not safe to conjecture what its equivalent is beyond where the field work has been carried in detail. The back slope of this escarpment, which is comparitively even, is spoken of locally as a limestone prairie in contrast to the sandstone area to the east.

THE HOWARD ESCARPMENT.

This is a low, even escarpment which from a distance somewhat resembles artificial embankments. It is seen at Howard in the north part of town. It is produced by thin limestone capping a shale bed which weathers very easily. Riding on the railroad, one can see it very conspicuous on the west side of the track from Moline to Severy. From this place to Climax the road cuts off a portion of it to the east. Beyond Climax it is again seen west of the railroad to Fall River. South of Moline its course is indicated on the map as being to the west of Middle Cana. At Wauneta it is somewhat higher and produces the peculiar rounded hills near that place.

THE EUREKA ESCARPMENT.

This escarpment is very conspicuous at Eureka. The town lies in the valley of Fall River, the escarpment making a high wall to the north, west and south. The shale bed in the face of the escarpment carries some coal at various places, and the limestone above the shale has been traced in detail to the south, and is found to be persistent though not very heavy. It would appear from a hasty reconnoisance that it caps the terrace which is prominent just west of the railroad from Eureka to Madison and at the latter place. From Eureka southward it runs on a sinuous line around the head of Honey creek, Tadpole creek, is prominent on Otter creek where the north and south branches unite, is found halfway between Severy and Piedmont, passes to the west of Pawpaw creek and is prominent on Elk River about five miles west of Howard. West of Moline it is the first hill beyond the low ones in the edge of town which belong to the Howard escarpment. It has been traced to Leeds, thence south, passing west of Grant creek and bending in an irregular course, producing the east bluff of the Cana at Cedarvale and for some distance north. To the south of Cedarvale it passes around the head of Rock creek and to the state line.

THE REECE ESCARPMENT.

This escarpment has not been traced in detail. It is present at Reece as the most conspicuous topographic feature and was seen at a distance in doing other field work between there and Grenola. It runs approximately parallel with the Eureka escarpment and about six miles to the west of it, but gradually approaching nearer to it southward. Half way between Moline and Grenola it may be seen to the north, forming the high hills. It then curves to the north around the head of Big Cana and blends with the eastern slope of the Flint Hills west of the creek.

UPPER LIMIT OF THE CARBONIFEROUS.

The Cottonwood Falls limestone and the bed of shales above constitute the upper member of the Carboniferous. The line of outcrop of this formation has not been traced. Prosser* has identified the formation west of Reece, Grenola and Cedarvale. The line shown on the accompanying map as the limit of the Carboniferous is therefore only approximately correct. This formation does not produce a conspicuous escarpment, and the limestone is masked in the eastern slope of the Flint Hills.

CORRELATION WITH THE PHYSIOGRAPHIC FEATURES IN MISSOURI.†

The blending of two or more escarpments, or *vice versa*, the splitting up of an escarpment into two or more, as well as the total disappearance of others, make it appear that if the same conditions hold in Missouri that we find in Kansas, there can be little certainty that any escarpment will continue across the two states.

The Cherokee Lowlands are the equivalent of the Nevada Lowlands. The Cherokee Lowlands extend as a belt across the corner of Kansas. The Nevada Lowlands are a continuation of this belt into Missouri, where the area narrows to a point according to the mapping by Marbut.

South of Fort Scott the Pawnee and Oswego escarpments blend. North of the Marmaton River the escarpment thus formed passes into Missouri. I judge that it is this escarpment which, after a short curve to the east, continues northward to the Osage river, and is the one described by Marbut as entering Missouri at that place, and named by him the Henrietta escarpment. This escarpment is considered by him as the western border of the Nevada Lowlands,

*Kansas Univ. Quart., Oct., 1897.
†Marbut, Geol. Sur. Missouri, Vol. X.

just as the Oswego escarpment is the limit of the Cherokee Lowlands. Its course in Missouri is rather an unexpected one to me, since it seems to cut across to the eastern border of the Carboniferous.

The Erie escarpment has been traced northward by Mr. Bennett to where it passes over the state line at the northeast corner of Linn county. It is probable that this escarpment continues in a sinuous course in Missouri around the head of some streams and is the same one described by Marbut as the Bethany Falls escarpment which he states enters Missouri in the southern part of Cass county. The region between the Henrietta and Bethany Falls escarpment has been called by him the Warrensburg platform. To the west of the Bethany Falls escarpment Marbut describes the Lathrop platform and the Marysville Lowlands. It is not possible here to give the Kansas equivalents of these belts, since a large area intervenes which I have not studied. It would appear; however, that in Kansas the escarpments described, or others similar, continue to the Missouri river.

Variations of External Appearance and Internal Characters of Spirifer Cameratus Morton.

Contributions from Paleontological Laboratory, No. 30.

BY J. W. BEEDE.

With Plate VI.

GENERAL DESCRIPTION.

Shell medium to large in size, greatest convexity back of middle, variable in outline from sub-semicircular to trigonal; anterior margin sharply rounded to truncate-sinuate, lateral margins slightly curved to nearly straight, pointing outward and backward to the ears; hinge line equalling the greatest width of the shell, sometimes prolonged into attenuate ears; cardinal area broad, extending to extremity of hinge line; foramen broadly triangular and nearly equilateral, partly closed in upper part by pseudodeltidium; beak high, prominent, somewhat recurved over cardinal area which is sometimes slightly arched; mesial sinus prominent, beginning at beak and broadening and deepening until the front margin is reached; fold of dorsal valve to correspond and beak of the same moderately rounded beneath that of the other valve; interior of ventral valve marked by a sub-elliptical muscular impression in the vicinity of the beak, posterior end of this depression extending to the hinge line or beyond, bisected by an indistinct, mesial ridge, radiating from which are small indistinct ridges for the attachment of the muscles. The cardinal area projects, leaving a large space beneath; the shell is here well pitted for attachments; a small tooth and depression are developed on the inner corners of the cardinal area. Shell of dorsal valve thin, muscular marking indistinct, hinge line at beak broadly and shallowly arched, one prominent socket on each side of arch, for teeth of other valve, two small elevations in the center for attachments. The exterior markings of the shell consist of rather large, bifurcating, radiating striæ or costæ, almost always fasciculated, covering the entire shell to the

tips of the ears where they are nearly parallel. There are in unweathered, unworn specimens minute pustules arranged in somewhat radiating order, also lines of growth sometimes visible on front border. In exfoliated specimens the fasciculation is less distinct. Measurement of average specimen: length, 32 mm.; breadth, 42 mm.; convexity, 21 mm.

VARIATIONS.

The variation in outline is great, as illustrated in figures 2 and 17, in Plate VI, and intermediate forms, the former having the hinge line extending into mucronate ears, while in the latter the hinge line is shorter than the greatest width of the shell. The difference in convexity of the shells is shown fairly well in figures 5, 6, 7, 23, Plate VI, contrasted with 1, 8, 21, 22. The mesial fold is often sharper in the less ventricose forms.

This species has been considered by some to be identical with *S. striatus* (Martin) Davidson. Schuchert has pointed out a difference between the two which is persistent,* and a marked difference will be found on comparing with figure 9 of Plate VI, the spires of the two species, as shown in the accompanying figure (1), which is reproduced by Mr. Prentice from Schuchert's figure in Eastman's Translation of Zittel's Hand Book of Paleontology, p. 386, A. In *S. striatus* the spire is long, loose, and acute at its apex, while in *S. cameratus* the spires are short, compressed, and very obtuse and inclose a slightly smaller angle. Even allowing a considerable amount of variation to each species there would then be ample distinction between the two; furthermore, the compressed ears of the attenuate form of *S. cameratus* would hardly permit of so long a spire.

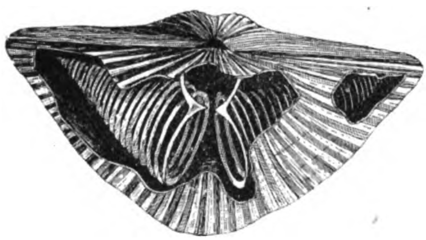


Fig. 1.

The variation of the internal characters of these species is quite as remarkable as that of the external portion. The two extreme forms will serve to illustrate the great range of these variations. In some specimens the muscular scar in the ventral valve is elliptical and extends beneath the beak and floor of the foramen, which is thick and plate-like; the teeth are not supported by heavy de-

*Bull. U. S. G. S. 87, p. 384: "The latter species (S. striatus), however, is closely and finely reticulated with concentric growth lines, while in S. camreatus the plications are covered with small pustules which are arranged in radiating lines."

posits of shell or thick plates, and the cavity formed by the jutting of the cardinal area is one general posterior cavity, extending across the shell, only slight ridges being present behind and below the teeth. This form is illustrated in figure 2.

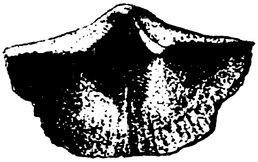


Fig. 2.

In other specimens the posterior portion of the shell is much thickened; muscular depression not extending beneath the beak or hinge line, that portion of the cavity being filled with shell, which has encroached on the muscular area, until it is small and nearly circular; the small mesial ridge is much less distinct than in the previous specimen. This thick deposit of shell also forms a heavy support for the teeth, and divides the posterior cavity into a right and left cavity. See figure 3.

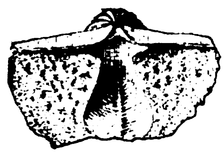


Fig. 3.

A minute, young specimen figured in the plate (figure 31) is nearly destitute of markings. The mesial fold and sinus are present, but have no indication of striæ upon them; there are indications of the three plications on each side of the fold; no other markings visible. Cardinal area visible, but ill-defined, and the entire shell has a glossy indefiniteness about it. The following measurement of specimens will illustrate the great variation in form of this species:

	Length.	Breadth.	Convexity.
	30 mm.	60 mm.	20 mm.
	50	31	20
	50	35	28
	55	37	23
	14	18	8
Average form.....	32	42	21
Very young.......	4	3	2½

One would naturally look for the degree of deposition of shell material in the ventral valves of the above specimens to be proportionate to the age of the shell, but from the material at hand I am able to draw no such inferenca.

Apparatus to Facilitate the Processes of Fixing and Hardening Material.

BY WILLIAM C. STEVENS.

The following method of carrying material through the processes of fixing and hardening has been of great utility in dealing with very small objects, such as root tips, sporangia, and young flower buds, and in keeping separate material of successive stages of development. The method is also useful in the hands of students, since it economizes space and material and otherwise facilitates processes which are always difficult to the beginner.

The material to be fixed and hardened is cut in the smallest pieces compatible with the purpose of the study, and then put into cloth-bottomed glass buckets which are made as follows: Cut pieces 3 cm. long from glass tubing of about 1 cm. in diameter. To do this uniformly and easily provide a guage block, as shown in Fig. 1. A hole, a, is bored in the block close to the bottom, just large enough to admit the tube easily and with a depth about 2 mm. less than the length of the piece to be cut off. Nail on the guage strip, b, so that when the glass cutter is held against it the cutting wheel will stand directly over the glass tubing when inserted in the hole for cutting off. With the fingers of one hand placed against the face of the block which is opposite the hole, press the glass cutter with the thumb against the guage strap and down on the tubing. Then turn the tubing with the other hand until the cutter has encircled it, but no more, for the cutting wheel will be injured by retracing the rough surface of the cut already made. Withdraw the tubing from the guage and at one place deepen the cut with a file. To separate the piece thus marked off grasp the tubing with both hands, with the thumbs opposing the fingers, as one would

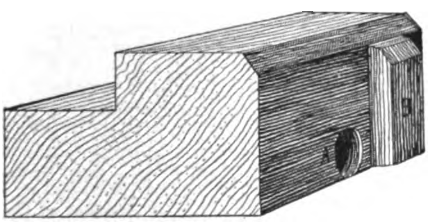


Fig. 1.

naturally do in breaking a stick, taking care to have the thumbs meet on the circle made by the glass cutter directly opposite the deeper cut made by the file. Break off the piece with a strong longitudinal as well as lateral pull. The piece of tubing is completed for use by turning outward the rim of one end and then heating the other end in the flame of a Bunsen burner sufficiently to melt away the rough edge. This is accomplished in the following manner: Whittle down one end of an electric light carbon so that it will fit snugly into the piece of glass tubing, and thus serve as a handle while the free end is being held in the Bunsen burner flame. Whittle down one end of another carbon to a blunt pencil point which is to be used in bending outward the rim of the tubing. Now hold one end of the piece of tubing in the flame by means of the carbon handle and turn it slowly so that the edge may be heated to glowing evenly all around. Then press the carbon pencil point into the softened end of the tubing, at the same time giving the carbon a twist so that the edge may be turned out evenly. It is best to turn out the rim only sufficiently to form a shoulder for tying on the cloth bottoms, and accordingly only the extreme end of the tubing should be held in the flame, and the pressure of the carbon point should be moderate. The other end of the tubing is next held in the flame and slowly turned until the edge is melted smooth, but not to such an extent that the rim begins to draw inward. The upper half of the tubing may now be ground on an emery wheel or grindstone so that it may be written on with a lead pencil. Tie a piece of thin muslin with a thread over the bottom of the tubing and trim the free edge close so that it may as little as possible soak up and carry reagents from one receptacle to another when the buckets are in use. In tying on the cloth have one end of the thread extend about 8 cm. from the knot. Make a hole with a needle at the center of the cloth bottom and pass the thread up through this. The buckets are to be suspended in the reagent bottles by means of this thread. The completed bucket is represented by Fig. 2.

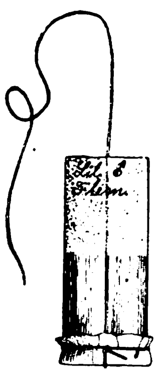


Fig. 2.

If the material to be fixed contains so much air that it will not sink in the fixative, and an air pump is not available, the material may be made to sink by stringing on the thread a porcelain or glass button, or better a solid

glass bead large enough to cover the mouth of the bucket, and then submerging the bucket in the fixative. But if an air pump can be had it is better to stopper the bucket with a cork which is trimmed flat on two sides so that the air may escape at the top as well as through the cloth bottom. Then the bucket should be placed in a bottle of water, or better,.5 per cent chromic acid, from which the air is then pumped, and after the atmospheric pressure is again turned on the material will immediately sink. The water or chromic acid should then be replaced at once with the intended fixative. If the material is in exceedingly small pieces, and would be likely to escape through the crevices left unstopped by the cork, it will be best to cork the bucket tightly and allow the air to escape only at the bottom. To wash out the fixative the buckets may be stoppered with corks sufficiently large to buoy them up, and then they may be floated in a tumbler of water into which water is kept slowly running; or the buckets may be suspended in the water by fastening their threads into notches in a stick placed across the tumbler. After the fixative is washed out the corks should be discarded and the buckets suspended in the different grades of alcohol, etc., to about two-thirds of the depth of the bucket. Reagent bottles, conveniently arranged for suspending the buckets, may be prepared as shown in Fig. 3. The bottles are ordinary mustards, and the sticks for fastening the thread have been notched at the ends for the thread and grooved at the center to receive the rubber band which fastens them to the neck of the bottle. We find it convenient to notch and groove the piece of wood from which the sticks are prepared before the sticks are cut off. The notches may be run with an ordinary marking guage, and the groove may be made and the sticks cut off with a saw in a miter box. A guage strip should be clamped to the back of the miter box so that the sticks may be quickly cut to a uniform width.

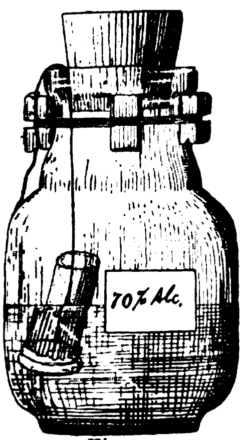


Fig. 3.

The material may remain in the buckets until brought into the paraffin on the oven, or even throughout the time it remains on the oven and until it is ready to be poured out for imbedding in the cold paraffin. I sometimes find it convenient to leave very small objects in the bucket on the oven until the solvent of the paraffin is evaporated, and then to lift the bucket quickly and plunge it

into cold water. At any time thereafter the bucket may be slightly warmed after cutting off the cloth bottom and the plug of paraffin pushed out. The plug may then be shaped up and mounted on the microtome. This method is particularly useful where sporangia or small buds of different ages are prepared with the object of getting sections in different directions and of different ages at each cut of the microtome knife. The buckets will also be found useful in preserving material in alcohol separated and labeled for future use.

The Preparation and Use in Class Demonstration of Certain Cryptogamic Plant Material.

BY MARSHALL A. BARBER.

The following is a description of some methods in use in the laboratory of Cryptogamic Botany in the University of Kansas. Some of these, in various forms, perhaps, may be now in use by many teachers; but they are described in the hope that they may be in part new, if not to all, at least to some who are engaged in teaching Cryptogamic Botany.

For demonstrating the evolution of oxygen by Algæ, and the relation of Bacteria and Infusoria to this gas, the following apparatus is used: A drop of water, or nutrient substance, containing the Algæ, Bacteria and Infusoria is placed in the center of a large cover glass. A smaller cover is then placed over the drop and the whole sealed, smaller cover down, to a ring cemented to a slip or any of the devices ordinarially used in making hanging drop cultures. There should be just liquid enough to fill the space between the covers, and the smaller cover should, of course, be less in diameter than the ring, so that a dry area will separate the liquid under the cover from the cement. The preparation is thus enclosed in a moist cell, and may be studied for days. Observations may be made with lights of different colors, different gases may be introduced into the cell, and the changes in the organisms due to these factors or to variations in their nutrient medium may be readily noted.

In obtaining plasmodia for the study of Myxomycetes during the winter months I have had very good success with sclerotia. Sclerotia of one or two forms are rather abundant in this region, and are usually found in rotten wood or on the ground under old logs. Pieces of this material put in a moist, warm place usually develop plasmodia in a few hours; and the plasmodia may be nourished and made to fruit or allowed to return to the resting form. I have used rotten wood and fleshy fungi for feeding plasmodia. Small plas-

modia for the observation of protoplasmic currents may be obtained by putting pieces of sclerotia in a hanging drop of water or some nutrient fluid enclosed in a cell, by transferring a piece of the living plasmodium to the cell, or by placing a large cover glass on a plasmodium and transferring it to a cell after the plasmodium has run over it. These small plasmodia will adhere to cover glasses and may be put into fixing and staining solutions without removing them. I have had good success in obtaining swarm spores of Myxomycetes by sowing large numbers of spores in vials of water. The germination of the spores cannot be followed so well as in drop cultures, perhaps, but large quantities of swarm spores may be obtained for class use.

In the study of the zoöspores and early stages of some Algæ I have used drop cultures prepared as follows: Incisions are made with a sharp knife in a piece of cork, and large cover glasses are fitted edgewise into the cuts in such a way that, when the cork is floated on water, some cover glasses are entirely submerged, others only partially. If the apparatus be placed in culture dish or pool containing Algæ, zoöspores of some kinds will fasten to these cover glasses and grow. The cover glasses, with germinating spores and young plants, may then be mounted over cells and studied. This method shows well the formation of zoöspores by young plants, as occurs in Oedogonium, and the way by which the plants fasten to their support. In the half submerged cover glasses the organisms which grow at the very surface of the water may be obtained, since the cover glasses, fixed to a floating object, remain at the same depth.

Few fungi are more favorable for the study of the formation of zoöspores than the Saprolegnieæ. Their rapid development and the ease with which they may be mounted and observed make them good material for the study of the growth of hyphæ also, and the formation of oögonia, oöspores and antheridia. An abundance of this material may be obtained, as is well known, by throwing insects or pieces of other organic material into water brought from a pond or stream; but difficulties often come from the too great multiplication of Bacteria in the culture dishes. I have had very good success in avoiding this trouble, by keeping a liberal supply of water plants, especially such as Elodea and Myriophyllum in the cultures. I find that when these plants are used there is no need of changing the water, and several insects may be put into a crystallizing dish of average size without the cultures becoming foul, especially if the room is not too warm. I have found spiders

among the best kinds of insects to use, and they are allowed to sink until they rest on the water plants beneath the surface. It is, of course, advisable to have the water free from animals which may eat the insects.

It is often desirable to demonstrate to large classes the methods by which spores are scattered in mosses, ferns, Equisetums and other plants in which hygroscopic action is observed. For this purpose the stereopticon has been used in the following way: Rather thick sections are made through the fruiting parts where ferns and Equisetums are used. These are mounted in water on a slip without a cover, the surplus water is drawn off and the preparation is placed in the ordinary apparatus for projecting objects on the screen. The heat from the source of light causes the water to dry rapidly, and just at the moment of drying the characteristic movements of the annuli and elaters may be seen projected on the screen. The addition of moisture will cause the hygroscopic parts to resume their former position; in the case of Equisetum, breathing on the side is sufficient. To observe the action of the peristome of moss capsules the whole capsule is mounted.

More exact methods are doubtless often more useful in investigation, and other ways of applying the ones given above may suggest themselves to teachers. These, however, have been found very helpful in teaching, especially in classes of beginners; and in the form given above have proved successful under the conditions present.

Lantern or Stereopticon Slides.

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VOL. VII. JULY, 1898. No. 3.

Individual Variations in the Genus Xiphactinus Leidy*

Contributions from the Paleontological Laboratory, No. 31.

BY ALBAN STEWART.

With Plates VII, VIII, IX, X.

The exceptionally fine collection of *Xiphactinus* material in the Kansas University museum has suggested a series of comparisons which show that the individual variations within the species are both numerous and interesting. My attention was first called to these facts by finding certain individuals which I could not place in any of the known species from America with any degree of satisfaction, and yet the differences were not sufficient to be called specific. The comparisons are confined entirely to the jaws, and show differences in size, shape, and arrangement of the teeth that are very marked, and I think prove conclusively that these characters can not be regarded as specific in the two species involved.

For convenience to those who may wish to examine the material in the future, the catalogue numbers on the cuts of the specimens will be used in reference to each individual in this article. As the relation of *X. molossus* and *X. thaumas* are most affected by this discussion, the original description of the jaws as given by Cope,† will be partially repeated here in order to have the specific characters of these parts well in mind.

Xiphactinus audax Leidy (Proc. Acad. Nat. Sci., Phila., 1870, p. 12) has been shown to be a synonym of *Saurocephalus* Cope (U. S. Geol. Surv., Wyoming, etc., 1872, p. 418). In a letter to Prof. Mudge, dated October 28, 1870, which will shortly be published in the fourth volume of the Kansas University Geological Survey, Cope refers it to *Saurocephalus thaumas* (*Portheus thaumas* Cope). After carefully comparing the description and figure of the pectoral spine of *X. audax* I was led to the same conclusion; and as the genus *Portheus* was not made known by Cope until 1871 (Proc. Am. Phil. Soc., 1871, p. 173), according to the rules of nomenclature *Xiphactinus* should have priority.

†Cret. Vert., 194-95.

Xiphactinus molossus Cope.†

"The *premaxillary* is vertically oval, convex externally, nearly flat within, and more than half underlaid by the anterior lamina of the maxillary. The anterior or median margin is regularly convex, and exhibits no surface or suture for union with the bone of the opposite side. Its posterior margin extends obliquely backward to beneath the superior articular condyle of the maxillary, and has a ragged edge, though the suture is squamose. Its superior margin is deeply inflected in front of the condyle, and then convex and thickened. The anterior margin is thick and rugose with tubercular exostoses. There are but two teeth, which are very large, and directed obliquely forward; the first is two-thirds the diameter of the second.

"The *maxillary* is a large laminiform bone, with the upper margin considerably thickened proximally, but much thinned distally. It is abruptly contracted at the distal two-thirds of its length, apparently for the attachment of a supernumerary bone. The extremity is curved saber-shaped upward, and has an acute toothless edge. The teeth are: four small, five large, and eighteen small. These teeth, except the largest, have cylindric bases; the crowns (and bases of the latter) are slightly compressed or oval; they are straight and regular, and lean backward. The middle one of the five is largest, being six times as long as the smaller ones, but little more than half as long as the large premaxillary or mandibular.

"The *mandibular rami* are short and deep, and have but little mutual attachment at the symphysis. They are not incurved at that point, and were bound by ligament only. There is no coronoid bone, and the articular is distinct. * * * The teeth are as follows: Two large—a transverse groove; three large; four very small; nine medium; and two very small—total, twenty. These teeth have straight cylindric crowns, with cementum without striæ or facets. The larger are little compressed."

Xiphactinus thaumas Cope.*

"The *premaxillary* is an obliquely oval or subpentagonal bone, the suture with the maxillary is not toothed, and the anterior or free edge is smooth, not tubercular as in the two specimens of *X. molossus*. There are but two teeth, of which the anterior is immense, and the second little more half its diameter. The maxillary is stout and supports in front four very small teeth; then three very large, of which the median is largest.

*Cert. Vert., p. 197.

"The *dentary* is similar in form to that of *P. molossus*, but rather more numerous teeth. Counting from the front, there are two large, one rather small; two large, and eighteen small and medium following; the smallest from third to ninth, inclusive. The alveolæ are nearly round."

Below is given a table showing the arrangement of the teeth in the specimens under consideration:

MANDIBLES.

No.	Medium.	Large.	Small.	Large.	Small.	Large.	Small	Medium and Small.	Medium.	Small.	Total.
1,	..	2	..	3	..	..	5	..	9	1	20
2,	..	2	1	1	..	..	8	12	..	..	24
3,	..	2	..	3	..	..	5	..	2+	..	12+
4,	..	2	..	2	2	..	4	..	10	2	22
88,	..	2	..	3	..	..	3	..	10	4	22
127,	..	2	..	3	..	..	6	..	10	..	21
135,	..	2	..	3	..	..	7	..	9	1	22
155,	..	2	.:	1	2	1	..	11	..	2?	18
179,	..	2	1	1	..	..	8	11	..	..	23
275,	..	2	1	3	..	..	..	..	..	..	
279,	..	2	..	3	..	..	..	..	..	..	
287,	..	2	..	3	1	..	4	..	9	3	22
314,	..	2	..	4	..	..	8	..	8	2	24
353,	2	..	..	2	..	..	..	17	..	..	21

	MAXILLARY.					PREMAXILLARY.			
No.	Small.	Medium.	Large.	Small.	Total.*	Small.	Large.	Small.	Total.
1,	2	..	5	..	7+	..	2	..	2
2,	4	..	5	22	31	..	2	..	2
3,	4	..	5	23	32	..	2	..	2
4,	4	..	5	27	36	..	2	..	2
88,	..	..	4	23	27	1	1	..	2
132,	1	..	5	20	26	..	..	..	..
155,	..	1	5	..	6+	..	2	2	4
179,	5	..	4	26	35	..	2	..	2
266,	2	..	6	19+	27+	1	2	..	3
275,	2?	..	5	25+	32+	..	..	..	..
279,	2	..	4	18	24	..	3	..	3
287,	3	..	5	26	34	..	..	..	..
301,	5	..	3	15	23	..	..	..	..
353,	3	..	4	22?	29?	..	2	..	2

From the above table it will be seen that in no instance do the size and arrangement of the teeth exactly agree with either *X. molossus* or *X. thaumas.* The number of teeth is also as varied as the size and arrangement, thus: in No. 88, the number of maxillary teeth corresponds with *X. molossus* but the mandible contains

two more than in this species. In No. 179, the mandibular teeth correspond in number with *X. thaumas*, but as the exact number of maxillary teeth is not given by Cope in his description of this form, the agreement in number of teeth on these parts can not be determined. No. 1 corresponds with *X. molossus* in number of mandibular teeth, but as the number of maxillary teeth of this specimen is somewhat in doubt, the agreement of these parts is uncertain. Three more or less constant characters are observed in the arrangement of the teeth in all the specimens under examination, viz: the four small anterior teeth on the maxillary, the two large teeth on the premaxillary, the two large anterior teeth on the mandible. The last of these seems to be the most constant of the three. The next in variation is the number of teeth on the premaxillary, and the one of greatest variation is the number of small anterior maxillary teeth.

From the above it will be seen that the size and arrangement of the teeth on the maxillary and mandible cannot be taken as specific in either species. The premaxillary teeth show diversity of character in size and arrangement quite as marked as the maxillaries and mandibles. In some instances the teeth are about equal in size, in others there is a noticeable difference, as in No. 301, where the posterior is very much smaller than the anterior, and in 88, where the opposite is the case.

Some of the specimens possess three premaxillary teeth. In No. 266 the anterior is very small and the two posterior large, in No. 279 there are three of about equal size, and in 155* four, the two anterior of which are large and the two posterior very small.

Below is given a table showing the great variety in size of the specimens:

	MANDIBLE		MAXILLARY.	
No.	Length of alveolar border.	Depth at coronoid.	Length from premaxillary.	Depth at condyle.
1,	242.5	145.5	.....	102
2,	251	144	260+	111
3,	279	140	314	119
4,	194	109	216.5	78
88,	260	136	273.5	92
127,	.....	.....	.....	88.5
132,	.....	.....	264	98
135,	197	.....	.....	.....
155,	215	.....	205+	83.5
179,	185	94	183	68
266,	.....	.....	.....	84
275,	.....	.....	220+	94
279,	.....	.....	263	90
287,	210	.....	220	80
301,	.....	.....	.....	90
314,	242	130	.....	.....
353,	195	115	219+	75

*It would be well to mention in this connection that No. 155 possesses characters other than number, size, and arrangement of the teeth, that separate it from *X. molossus* or *X. thaumas*, the most important of which are the articulation of the maxillary and pre-maxillary, the difference in form of the palatine condyle, and the greater depth of the dentary with reference to its length. It probably represents *X. lestero*, although this fact I have not yet definitely determined.

The variation in size is as marked as the other differences mentioned above, thus: the length of the maxillary from the premaxillary varies from 183 mm. to 325 mm., giving a difference of 142 mm. in the length of the alveolar borders of the mandibles of these two specimens, Nos. 3 and 179; there is a difference in length of 98.5 mm. The transverse groove mentioned by Cope* as one of the characters of *X. molossus*, varies greatly in development. In some specimens it is very prominent, in others scarcely noticeable. The tubercular exostoses on the anterior margin of the premaxillary vary quite as much as the other characters enumerated; they probably increase in development with the age of the animal. However, this, with the presence or absence of a ragged suture between the maxillary and premaxillary would not affect the specific position of either form under consideration. Other differences in outline, etc., are noticeable in the figures and are not worthy of mention here.

In the light of the above facts it would seem that *X. molossus* and *X. thaumas* are synonymous, and as Cope has admitted that *X. thaumas* and *X. audax* are synonyms,† this form should be known in the future as *Xiphactinus audax* Leidy.

*l. c. 19.
†l. c.

Lawrence, Kas., March 10, 1898.

A Geological Reconnoisance in Grant, Garfield and Woods Counties, Oklahoma.

BY GEO. I. ADAMS.

With Plates XI and XII.

In March, 1898, the writer spent some time in a general reconnoisance of Grant, Garfield and Woods counties in northwestern Oklahoma. The accompanying map (Plate XI) shows the geology south of central Kansas, and records such information as I was able to obtain concerning the adjacent portion of the territory.

The main geological formation in the above mentioned counties is the Red Beds. This is overlaid by an irregular Pleistocene deposit, which is apparently not over fifty feet thick. The latter is not shown on the map. The Red Beds, the age of which is still undetermined because of their barrenness of fossils, are either Permian or Triassic. They have been studied in Kansas and a summary of the literature can be found in the Kansas University Geological Survey, Vol. II, in the chapter by C. S. Prosser, on The Upper Permian. Following his description I give here the general characters of the formation. The Red Beds, or Cimarron series, consist principally of red sandstones and shales, some of which, when wet, are of a bright red or vermilion color. The sandstones are soft and friable while the shales are arenaceous or argillaceous. Thin layers of grayish to greenish gray sandstones, and grayish spots are not of unfrequent occurrence. Near the middle portion of the terrain the shales contain considerable deposits of salt. Above the salt shales are variegated shales and sandstones, in which are thin layers of satinspar, selenite and other forms of gypsum. Capping these shales is the main mass of gypsum which is a conspicuous ledge several feet in thickness. Succeeding the massive gypsum are bright red shales and sandstones that are more brilliantly colored. Gypsum is not so abundant in this upper portion of the Red Beds, but near the top is found a conspicuous stratum of magnesian limestone or dolomite.

The Red Beds. according to Prosser, may be considered as consisting of the lower beds or Salt Fork formation, the Cave creek

gypsum horizon, and the upper or Kiger formation. The gypsum horizon in Kansas has been mapped by P. G. Grimsley (Kan. Univ. Quart., Jan., 1897). On the accompanying map the continuation of this horizon in the western and southwestern portions of Woods county is shown. The country here described lies principally to the east of this horizon and is the formation geologically lower, i. e. the Salt Fork formation. The eastern boundary of the Red Beds lies beyond the limits of the counties here described and evidently trends to the east considerably from where it leaves the state line. From various sources of information I conclude that it passes to the east of Red Rock.

I entered the territory at the northwest corner of Grant county and proceeded to Wedford. The surface was at first undulating and the soil red, but it became more level and the soil graded into a black gumbo. The country is essentially a prairie land and the streams flow in valleys which are but little below the general level. Along the north side the Salk Fork, which flows in a sandy bed, there are low sand dunes in places. I next traversed the divide between the Salt Fork and the Cimarron, which is low and unbroken, except where the Red Beds are exposed in a line of "breaks" trending around the heads of the streams which are tributary to the Salt Fork from the south. Standing on the divide one sees the streams which flow to the north take their rise in small, rugged canons and emerge into broad valleys in a few miles, while the streams flowing southward into the Cimarron begin in meandering channels, which, in places, are bordered by low swells which are extinct sand dunes. The divide is migrating southward, and there are evidences of the heads of the small canons gradually capturing the drainage of the southward slope. The Red Beds, although not exposed except in the breaks as already spoken of, are everywhere found at a shallow depth in diging wells. The Pleistocene lies like an uneven mantle over these eroded surface. From inquiry I learned that the wells usually pass through it in less than fifty feet. One dug on the divide went through red soil and clay for twenty-five feet, then through fifteen feet of sand containing fine gravel. In the sand was found a large bone which was probably a femur of *Equus*. Another well passed through a layer of land shells at a depth of fifteen feet. The creeks are nearly all fed by springs at various places and there is little difficulty in obtaining water except near the breaks. The supply seems to be found in the Pleistocene or the upper surface of the Red Beds. I have seen springs of sweet water flowing from the Red Beds, in

other cases from the base of the sand hills or breaking from the Pleistocene. This is interesting when taken in connection with the fact that the water of the streams is nearly all brackish or saltish, and upon evaporation will leave behind a white efflorescence upon the vegetation or the dry bed of the stream. In the eastern portion of Woods county, just south of the Salt Fork, is situated the Salt Plain, which, with the encrustation upon the sand and salt grass, shimmers like a lake when seen in the sun. This Salt Plain has been considered by F. W. Cragin (Colorado College Studies, Vol. VI,) as being derived from the salt deposit of the lower portion of the Red Beds, as above described. Further west, in the vicinity of the gypsum horizon, the water in the creeks is bitter and often unusable for either man or beast, but wherever the sand hills or the Pleistocene deposits are sufficiently extensive sweet water is found in shallow wells or bursts forth in springs. On the south slope of the divide there are large areas covered with sand hills which are mostly extinct and bear a growth of blackjack timber. Along the north side of the Cimarron there is a belt several miles wide which is of this nature, but the sand hills are more active and the timber is absent or is replaced by cottonwood trees. Some of the hills reach a surprising height. Tall cottonwood trees are found growing upon them, and the shifting of the sand has occasionally buried the trees so that only the top branches are seen protruding in sand pits in the top of the hills. A ridge of sand hills near the mouth of Eagle Chief creek is fully 150 feet high. North of the Salt Fork, in the northeast part of Woods county, is a similar ridge.

South of the Cimarron river, in Woods county, the Glass mountains are marked on most maps of Oklahoma. A trip to this locality revealed the inaccuracy of the name. There is a bold escarpment with occasional isolated mounds, extending along the river and around the heads of the short creeks which are tributary to the river from the south. One of these mounds or buttes, perhaps the most prominent one, is situated at about the place at which the mountains are indicated on the map. From this place the escarpment is seen trending to the south and east away from the river, gradually becoming less prominent. Westward are many headlands and buttes which follow the course of the river. The protecting element in this escarpment is a ledge of gypsum twelve feet in thickness where I measured it. It is seen capping the butte in the illustration Plate XII. Below it are the readily yielding clays and soft shales of the Salt Fork formation. The butte here pictured

is separated from the main escarpment by a short gap. Its height above the lowest place in the gap is 150 feet, and its elevation above the river is considerably more. Just how far west along the Cimarron the gypsum is found I am unable to say. Cragin has reported it at Heman, near the county line (vide Prosser ibid, p. 91). I have indicated on the map by a dotted line its probable connection with the gypsum in Kansas.

The "breaks" which I have previously spoken of as trending around the heads of the tributaries of the Salt Fork from the south approach the river east of Alva several miles, so that at Alva the valley is narrowed to about three miles in width. West of Alva the country is more rugged, being broken by canons, and the clay and sandstones are seen sculptured into flat topped terraced hills. Near the west border of the county there is a prominent escarpment which I have seen from a distance. I am told that it contains a ledge of gypsum. Red Hill, shown on the maps of Oklahoma, is a portion of this escarpment. In drawing the broken line connecting the gypsum horizon in Kansas with the known portion in Oklahoma, I referred to this escarpment in so far as possible.

The Pleistocene deposists are not shown on the map, but are nearly everywhere present. They consist of clay and sand, mixed with a varying amount of gravel. The clay has a red appearance, as if derived from the Red Beds. The sand, however, is coarse and quartzitic, and the pebbles are frequently variable in size and very variable in character, being quartzitic, felspathic, granitic, etc., while occasionally I found cherts which contained fragments of fossils, such as might have been derived from a carboniferous limestone. Near the crest of the divide I have seen small areas of irregularly bedded sandstone which was loosely cemented. Near by the canons contained large gravel, left behind from the erosion of the Pleistocene. *Elephas* remains, although usually poorly preserved, are relatively abundant. As many as ten different finds being reported from the area here described.

Normal Forms of Projective Transformations.

BY H. B. NEWSON.

A linear fractional transformation in one variable, viz:

$$x_1=\frac{ax+b}{cx+d},$$

is always reducible to one or the other of the following normal forms:*

$$\frac{x_1-m}{x_1-n}=k\frac{x-m}{x-n} \quad \text{or} \quad \frac{1}{x_1-m'}=\frac{1}{x-m'}+\alpha. \qquad (1)$$

These two forms correspond to the two types of transformations of this kind, viz: transformations with two invariant points, and transformations with one invariant point. In these normal forms the constants of the transformations are the 'essential parameters'; each constant has a definite meaning.

In the first form m and n are the coordinates of the two invariant points and k is the anharmonic ratio of these invariant points and any pair of corresponding points x and x_1 in the transformation. In the second form m' is the single invariant point and α is a constant which characterizes the transformation. If p and p_1 are corresponding points, the normal form shows that

$$\alpha=\frac{1}{m'p_1}-\frac{1}{m'p}.$$

Some point q on the line is transformed to infinity; hence

$$\alpha=-\frac{1}{m'q}.$$

This linear fractional transformation may also be interpreted as a projective transformation of the lines of a flat pencil or of the planes of an axial pencil. The first type of transformation leaves two rays (planes) of the pencil invariant, and k is the anharmonic ratio of the pair of invariant rays (planes) and any pair of corres-

*See Klein's Elliptische Modulfunctionen, pp. 164 and 173.

ponding rays (planes). The second type leaves invariant a single ray (plane) m'. x and x_1 represent a pair of corresponding rays (planes), such that $\cot(x_1,m')-\cot(x,m')=\alpha$. If x' is the ray (plane) which is transformed into the perpendicular to m', then

$$\alpha=-\cot(x',m').$$

An analogous theory may be developed for the projective transformations of the plane and of space. The equations of the projective transformation in the plane are given in cartesian coordinates by

$$x_1=\frac{ax+by+c}{a_2x+b_2y+c_2} \text{ and } y_1=\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}. \qquad (2)$$

There are five types of such transformations, each type being characterized by its invariant figure. Corresponding to each type is a normal form in which the constants are the essential parameters of the transformation.

The equations of the projective transformations in space are given in cartesian coordinates by

$$x_1=\frac{ax+by+cz+d}{a_3x+b_3y+c_3z+d_3} \quad y_1=\frac{a_1x+b_1y+c_1z+d_1}{a_3x+b_3y+c_3z+d_3} \qquad (3)$$

$$z_1=\frac{a_2x+b_2y+c_2z+d_2}{a_3x+b_3y+c_3z+d_3}.$$

There are thirteen types of projective transformations in space; each is characterized by its invariant figure.* (See Fig. 1.)

The object of this paper is to give the normal forms of the five types in the plane and of the thirteen types in space. These forms have been arrived at from geometrical considerations rather than by purely algebraic processes. These forms are given for the most part without proof. The detailed proof is given for the first and second types in the plane; the other types are derived by analogous processes.

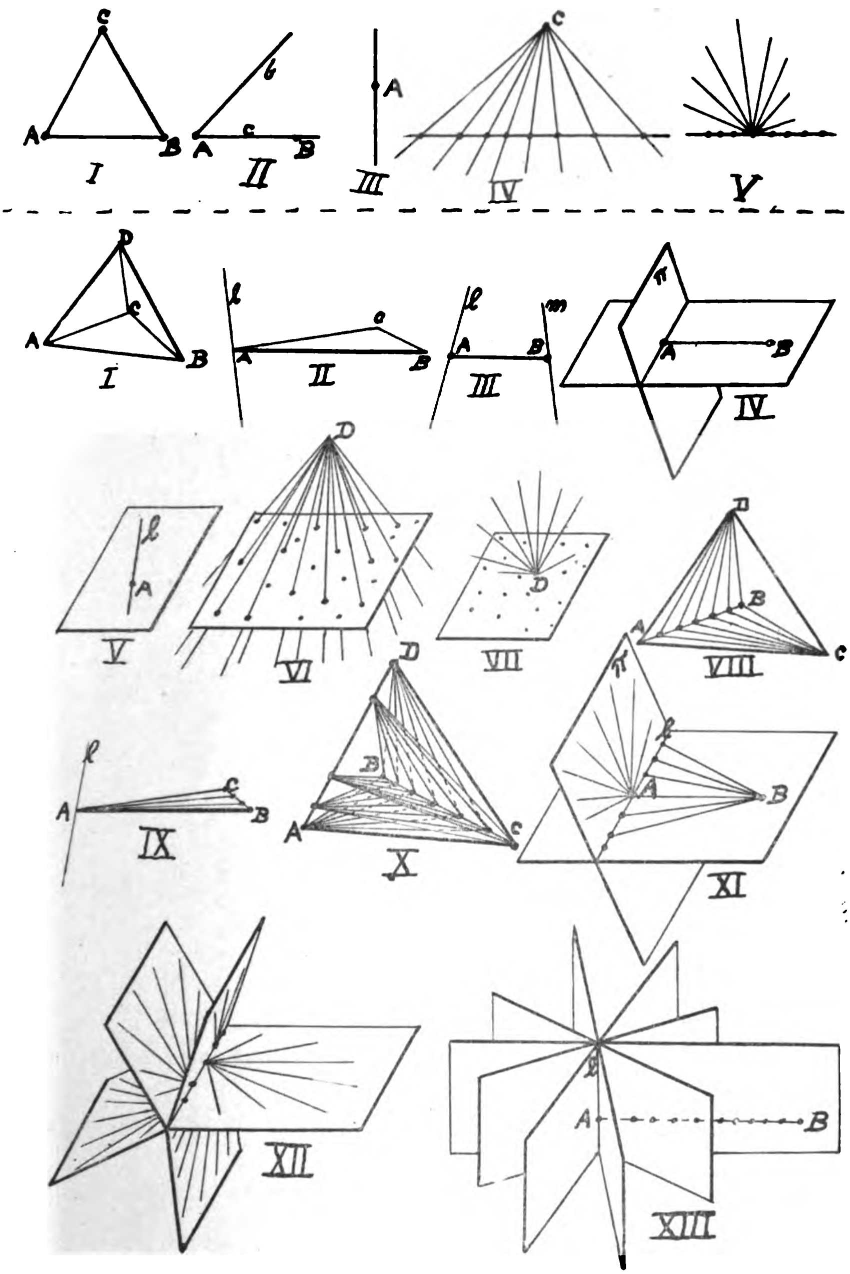
Part I.—The Plane.

TYPE I.

The most general form of projective transformation in the plane leaves invariant a triangle, the cartesian coordinates of whose vertices may be represented by $A(a,b)$ $B(a_1,b_1)$ and $C(a_2,b_2)$. In a previous paper† it has been shown that a projective transformation leaving a triangle invariant is characterized by the position of the triangle and three anharmonic ratios taken along the invariant

*See Kan. Univ. Quar., Vol. VI, p. 63.
†See Kan. Univ. Quar., Vol. V, p. 85.

Fig I.



lines or through the invariant points; and that the product of these three anharmonic ratios when taken in the same order round the triangle is unity. These three ratios are designated by k, k^{-c}, and k^{c-1}.

Let the point $P(x,y)$ be transformed to $P_1(x_1,y_1)$. The anharmonic ratio of the pencil $C(ABPP_1)=k$. The equations of the lines AC, BC, PC, P_1C are respectively

$$Y=\frac{b_2-b}{a_2-a}X+\ldots,\qquad Y=\frac{b_2-b_1}{a_2-a_1}X+\ldots,\qquad Y=\frac{y-b_2}{x-a_2}X+\ldots,$$
$$Y=\frac{y_1-b_2}{x_1-a_2}X+\ldots,$$

when X and Y are the current coordinates. Hence we have

$$\frac{\dfrac{y_1-b_2}{x_1-a_2}-\dfrac{b-b_2}{a-a_2}}{\dfrac{y_1-b_2}{x_1-a_2}-\dfrac{b_1-b_2}{a_1-a_2}}=k\frac{\dfrac{y-b_2}{x-a_2}-\dfrac{b-b_2}{a-a_2}}{\dfrac{y-b_2}{x-a_2}-\dfrac{b_1-b_2}{a_1-a_2}}.$$

Similarly the pencil $B(CAPP_1)=k^{c-1}$. Whence

$$\frac{\dfrac{y_1-b_1}{x_1-a_1}-\dfrac{b-b_1}{a-a_1}}{\dfrac{y_1-b_1}{x_1-a_1}-\dfrac{b_2-b_1}{a_2-a_1}}=k^{c-1}\frac{\dfrac{y-b_1}{x-a_1}-\dfrac{b-b_1}{a-a_1}}{\dfrac{y-b_1}{x-a_1}-\dfrac{b_2-b_1}{a_2-a_1}}.$$

These forms readily reduce to*

$$\frac{\begin{vmatrix}x_1 & y_1 & 1\\ a_2 & b_2 & 1\\ a & b & 1\end{vmatrix}}{\begin{vmatrix}x_1 & y_1 & 1\\ a_2 & b_2 & 1\\ a_1 & b_1 & 1\end{vmatrix}}=k\frac{\begin{vmatrix}x & y & 1\\ a_2 & b_2 & 1\\ a & b & 1\end{vmatrix}}{\begin{vmatrix}x & y & 1\\ a_2 & b_2 & 1\\ a_1 & b_1 & 1\end{vmatrix}};\qquad \frac{\begin{vmatrix}x_1 & y_1 & 1\\ a_1 & b_1 & 1\\ a & b & 1\end{vmatrix}}{\begin{vmatrix}x_1 & y_1 & 1\\ a_1 & b_1 & 1\\ a_2 & b_2 & 1\end{vmatrix}}=k^{c-1}\frac{\begin{vmatrix}x & y & 1\\ a_1 & b_1 & 1\\ a & b & 1\end{vmatrix}}{\begin{vmatrix}x & y & 1\\ a_1 & b_1 & 1\\ a_2 & b_2 & 1\end{vmatrix}}. \tag{4}$$

This form (4) is analogous to the first normal form of (1); for the latter may be written

$$\frac{\begin{vmatrix}x_1 & 1\\ m & 1\end{vmatrix}}{\begin{vmatrix}x_1 & 1\\ n & 1\end{vmatrix}}=k\frac{\begin{vmatrix}x & 1\\ m & 1\end{vmatrix}}{\begin{vmatrix}x & 1\\ n & 1\end{vmatrix}}.$$

*In *Continuierliche Gruppen*, pages 22 and 76, Lie gives results which, if carried a step farther, would lead to these forms.

Since the determinants in (4) stand for double the areas of certain triangles we have

$$k=\frac{\triangle P_1AC}{\triangle P_1BC}:\frac{\triangle PAC}{\triangle PBC}, \qquad k^{c-1}=\frac{\triangle P_1AB}{\triangle P_1BC}:\frac{\triangle PAB}{\triangle PBC}.$$

The normal form (4) may be expressed in terms of homogeneous coordinates by filling out the determinants with z, c, c_1 and c_2.

TYPE II.

The invariant figure of this type consists of two points A(a,b) and $B(a_1,b_1)$, the line AB, and another line ACC_1. Let the point P(x,y) be transformed to $P_1(x_1,y_1)$. This transformation is completely determined by the invariant figure and two essential parameters k and α, where k is the anharmonic ratio of the pencil $A(CBPP_1)$ and α is given by $\frac{1}{AC_1}=\frac{1}{AC}+\alpha.$

The equations of the lines CA, BA, PA, P_1A are respectively

$$Y=mX+\ldots, \qquad Y=\frac{b_1-b}{a_1-a}X+\ldots, \qquad Y=\frac{y-b}{x-a}X+\ldots,$$
$$Y=\frac{y_1-b}{x_1-a}X+\ldots$$

Whence we have

$$\frac{\dfrac{y_1-b}{x_1-a}-m}{\dfrac{y_1-b}{x_1-a}-\dfrac{b_1-b}{a_1-a}}=k\frac{\dfrac{y-b}{x-a}-m}{\dfrac{y-b}{x-a}-\dfrac{b_1-b}{a_1-a}}.$$

This form reduces to

$$\frac{\begin{vmatrix} x_1 & y_1 & 1 \\ a & b & 1 \\ 1 & m & 0 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & 1 \\ a & b & 1 \\ a_1 & b_1 & 1 \end{vmatrix}}=k\frac{\begin{vmatrix} x & y & 1 \\ a & b & 1 \\ 1 & m & 0 \end{vmatrix}}{\begin{vmatrix} x & y & 1 \\ a & b & 1 \\ a_1 & b_1 & 1 \end{vmatrix}}. \tag{5}$$

The equations of the lines C_1A, P_1B, and PB are written

$$Y=mX+b-am, \quad Y=\frac{y_1-b_1}{x_1-a_1}X-\frac{a_1(y_1-b_1)+b_1(x_1-a_1)}{x_1-a_1}$$

$$Y=\frac{y-b_1}{x-a_1}X-\frac{a_1(y-b_1)+b_1(x-a_1)}{x-a_1}.$$

The abscissa or the point C_1 is given by

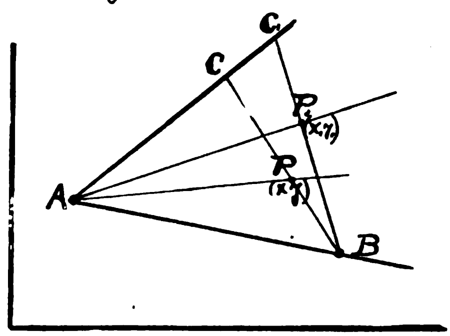
$$X_1=\frac{b_1(y_1-b_1)-(x_1-a_1)(b_1-b+am)}{y_1-b_1-m(x_1-a_1)}\ ; \quad \text{hence}$$

$$X_1-a=\frac{x_1(b-b_1)-y_1(a-a_1)+ab_1-a_1b}{y_1-b_1-m(x_1-a_1)}.$$

In like manner we find the abscissa of C and thence X—a,

$$X-a=\frac{x(b-b_1)-y(a-a_1)+ab_1-a_1b}{y-b_1-m(x-a_1)}.$$

Fig. 2.



From Fig. 2 we see that $\dfrac{X-a}{AC}=\dfrac{X_1-a}{AC_1}=\cos\theta$, where $\tan\theta=m$. Hence

$$\frac{1}{AC_1}-\frac{1}{AC}=\frac{\cos\theta}{X_1-a}-\frac{\cos\theta}{X-a}=\alpha.$$

Replacing X_1—a and X—a by their values we have

$$\frac{\begin{vmatrix} x_1 & y_1 & 1 \\ a_1 & b_1 & 1 \\ 1 & m & 0 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & 1 \\ a_1 & b_1 & 1 \\ a & b & 1 \end{vmatrix}}=\frac{\begin{vmatrix} x & y & 1 \\ a_1 & b_1 & 1 \\ 1 & m & 0 \end{vmatrix}}{\begin{vmatrix} x & y & 1 \\ a_1 & b_1 & 1 \\ a & b & 1 \end{vmatrix}}+\frac{\alpha}{\cos\theta} \tag{5a}$$

These two equations (5) and (5a) constitute the normal form of a transformation of type II. They may also be expressed in homogeneous coordinates.

TYPE III.

The invariant figure of type III consists of a single point A(a,b) and a line through it, as Al. Let P(x,y) be transformed to $P_1(x_1,y_1)$. The normal form of this type is given by the following equations:

$$\frac{\begin{vmatrix} x_1 & y_1 & 1 \\ a & b & 1 \\ m & -1 & 0 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & 1 \\ a & b & 1 \\ 1 & m & 0 \end{vmatrix}} = \frac{\begin{vmatrix} x & y & 1 \\ a & b & 1 \\ m & -1 & 0 \end{vmatrix}}{\begin{vmatrix} x & y & 1 \\ a & b & 1 \\ 1 & m & 0 \end{vmatrix}} + \alpha \qquad (6)$$

$$\frac{1}{\begin{vmatrix} x_1 & y_1 & 1 \\ a & b & 1 \\ 1 & m & 0 \end{vmatrix}} = \frac{1}{\begin{vmatrix} x & y & 1 \\ a & b & 1 \\ 1 & m & 0 \end{vmatrix}} + k\alpha \frac{\begin{vmatrix} x & y & 1 \\ a & b & 1 \\ m & -1 & 0 \end{vmatrix}}{\begin{vmatrix} x & y & 1 \\ a & b & 1 \\ 1 & m & 0 \end{vmatrix}} + \frac{k}{2}\alpha^2 + h\alpha. \qquad (6a)$$

The meanings of these constants or essential parameters are as follows: (a,b) are the coordinates of the point A; $m = \tan\theta$, where θ is the angle which Al makes with the axis of x. The transformation leaves invariant a pencil of conics all having contact of the third order at A; k is the reciprocal of the common radius of curvature at A of these conics, and $h|k$ is the cotangent of the angle which the line of centres of these conics make with Al; $\alpha = -\cot(x',l)$, x′ being the ray through A which is transformed into the perpendicular at Al.

TYPE IV.

Type IV represents a perspective transformation in which the centre A(a,b) is not on the axis of invariant points $y = mx + c$. Let P(x,y) be transformed to $P_1(x_1,y_1)$. Two conditions are to be satisfied; the first is that P, P_1, and A are collinear, the second condition is that the anharmonic ratio of the range $(ABPP_1) = k$, where B is any point on the axis.

The first condition is expressed by the equation

$$\frac{x_1 - b}{x_1 - a} = \frac{y - b}{x - a}. \qquad (7)$$

The second condition leads to the equation

$$\frac{\begin{vmatrix} x_1 & y_1 & 1 \\ a & b & 1 \\ 1 & m & 0 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & 1 \\ 0 & c & 1 \\ 1 & m & 0 \end{vmatrix}} = k \frac{\begin{vmatrix} x & y & 1 \\ a & b & 1 \\ 1 & m & 0 \end{vmatrix}}{\begin{vmatrix} x & y & 1 \\ 0 & c & 1 \\ 1 & m & 0 \end{vmatrix}}. \qquad (7a)$$

TYPE V.

Type V represents a perspective transformation in which the centre is on the axis. The two conditions to be satisfied are that A, P, P_1 are collinear and that $\frac{1}{AP_1}=\frac{1}{AP}+\alpha$.

The first condition leads to

$$\frac{y_1-b}{x_1-a}=\frac{y-b}{x-a} \tag{8}$$

The second condition is satisfied by the equation

$$\frac{1}{\begin{vmatrix} x_1 & y_1 & 1 \\ a & b & 1 \\ 1 & m & 0 \end{vmatrix}}=\frac{1}{\begin{vmatrix} x & y & 1 \\ a & b & 1 \\ 1 & m & 0 \end{vmatrix}}+\alpha. \tag{8a}$$

REDUCTION TO CANONICAL FORMS.

By a suitable choice of coordinate axes the five normal forms above may be reduced to their simplest or canonical forms.

If equations (4) be made homogeneous and the invariant triangle be taken for triangle of reference we get

$$\frac{y_1}{z_1}=k\frac{y}{z} \text{ and } \frac{x_1}{z_1}=k^{c-1}\frac{x}{z};$$

if the side z be made the line at infinity, these reduce to

$$x_1=k^{c-1}x, \text{ and } y_1=ky. \tag{9}$$

In like manner equations (5) and (5a) reduce to

$$\frac{x_1}{z_1}=k\frac{x}{z} \text{ and } \frac{y_1}{z_1}=\frac{y}{z}+\alpha; \text{ or } x_1=kx \text{ and } y_1=y+\alpha. \tag{10}$$

Equations (6) and (6a) reduce to

$$\frac{x_1}{y_1}=\frac{x}{y}=\alpha \text{ and } \frac{z_1}{y_1}=\frac{z}{y}+k\alpha\frac{x}{y}+\frac{k}{2}\alpha^2+h\alpha.$$

Interchanging y and z and then making $z=1$ we get

$$x_1=x+\alpha \text{ and } y_1=y+k\alpha x+\frac{k}{2}\alpha^2+h\alpha. \tag{11}$$

Equations (7) and (7a) reduce to

$$\frac{y_1}{x_1}=\frac{y}{x} \text{ and } \frac{y_1}{z_1}=k\frac{y}{z}; \text{ whence we have } x_1=kx \text{ and } y_1=ky. \tag{12}$$

Equations (8) and (8a) become

$$\left.\begin{array}{c}\dfrac{y_1}{x_1}=\dfrac{y}{x} \text{ and } \dfrac{z_1}{y_1}=\dfrac{z}{y}+\alpha; \text{ interchanging } y \text{ and } z,\\ x_1=x \text{ and } y_1=y+\alpha.\end{array}\right\}\quad(13)$$

The final forms in all cases agree with those given by Franz Meyer in the volume of papers read at the Chicago Congress, page 190.

Part II.—Space.

We come now to the consideration of the types of projective transformations in space. The normal forms of the thirteen types are determined from geometrical considerations. The expression of the results in determinant forms is not essential and indeed is sometimes a trifle strained. Each determinant when equated to zero is the equation of some invariant or otherwise essentially important plane connected with the invariant figure. Sometimes it is possible to express the equation of a plane in another form different from that here given and equally simple.

TYPE I.

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a_2 & b_2 & c_2 & 1\\ a_3 & b_3 & c_3 & 1\\ a & b & c & 1\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a_2 & b_2 & c_2 & 1\\ a_3 & b_3 & c_3 & 1\\ a_1 & b_1 & c_1 & 1\end{vmatrix}}=k\frac{\begin{vmatrix} x & y & z & 1\\ a_2 & b_2 & c_2 & 1\\ a_3 & b_3 & c_3 & 1\\ a & b & c & 1\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a_2 & b_2 & c_2 & 1\\ a_3 & b_3 & c_3 & 1\\ a_1 & b_1 & c_1 & 1\end{vmatrix}};\qquad \frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\\ a & b & c & 1\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\\ a_3 & b_3 & c_3 & 1\end{vmatrix}}=k^{1-r}\frac{\begin{vmatrix} x & y & z & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\\ a & b & c & 1\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\\ a_3 & b_3 & c_3 & 1\end{vmatrix}};$$

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a_3 & b_3 & c_3 & 1\\ a_1 & b_1 & c_1 & 1\\ a & b & c & 1\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a_3 & b_3 & c_3 & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\end{vmatrix}}=k^{1+r-rs}\frac{\begin{vmatrix} x & y & z & 1\\ a_3 & b_3 & c_3 & 1\\ a_1 & b_1 & c_1 & 1\\ a & b & c & 1\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a_3 & b_3 & c_3 & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\end{vmatrix}}.$$

The invariant tetrahedron is ABCD; the coordinates of A are (a,b,c); those of B are (a_1,b_1,c_1); those of C are (a_2,b_2,c_2); those of D are (a_3,b_3,c_3). k is the anharmonic ratio of the transformation along AB; k^{1-r} is that along AC; k^{1+r-rs} is that along AD.

TYPE II.

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_2 & b_2 & c_2 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\end{vmatrix}} = k\frac{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_2 & b_2 & c_2 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\end{vmatrix}}; \qquad \frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\end{vmatrix}} = k^{1-r}\frac{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\end{vmatrix}};$$

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\end{vmatrix}} = \frac{\begin{vmatrix} x & y & z & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\end{vmatrix}} + \alpha.$$

The coordinates of the point A of the invariant figure are (a,b,c); those of B are (a_1,b_1,c_1); those of C are (a_2,b_2,c_2); p and q are the ratios of the direction cosines of the line l. k is the characteristic anharmonic ratio along AB; k^{1-r} is that along AC; and α is the characteristic constant along l.

TYPE III.

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & q_1 & 0\end{vmatrix}} = k\frac{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & q_1 & 0\end{vmatrix}}; \qquad \frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a_1 & b_1 & c_1 & 1\\ a & p & q & 0\\ 1 & p_1 & q_1 & 0\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 2\\ a_1 & b_1 & c_1 & 2\\ 1 & p & q & 0\end{vmatrix}} = \frac{\begin{vmatrix} x & y & z & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\\ 1 & p_1 & q_1 & 0\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\end{vmatrix}} + \alpha;$$

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\\ 1 & p_1 & q_1 & 0\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & q_1 & 0\end{vmatrix}} = \frac{\begin{vmatrix} x & y & z & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\\ 1 & p_1 & q_1 & 0\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & q_1 & 0\end{vmatrix}} + \alpha_1.$$

The coordinates of the two invariant points A and B are respectively (a,b,c) and $(_1,b_1,c_1)$; the directions of the two invariant lines Al and Bl′ are determined by (p,q) and (p_1,q_1). k is the anharmonic ratio along the line AB; α is the characteristic constant along the line Al, and α_1 is that along Bl′.

TYPE IV.

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 0 & 0 & 1 & f \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 1 & p & q & 0 \end{vmatrix}} = k \frac{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 0 & 0 & 1 & f \end{vmatrix}}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 1 & p & q & 0 \end{vmatrix}} \qquad \frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 0 & q & -p & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 1 & p & q & 0 \end{vmatrix}} = \frac{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 0 & q & -p & 1 \end{vmatrix}}{\begin{vmatrix} x & y & x & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 1 & p & q & 0 \end{vmatrix}} + \alpha;$$

$$\frac{1}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 1 & p & q & 0 \end{vmatrix}} = \frac{1}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 1 & p & q & 0 \end{vmatrix}} + m\alpha \frac{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 0 & q & -p & 1 \end{vmatrix}}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 1 & p & q & 0 \end{vmatrix}} + \frac{m}{2}\alpha^2 + h\alpha.$$

The coordinates of the two invariant points A and B are respectively (a,b,c) and (a_1,b_1,c_1); the direction of the line Al is determined by p and q; f determines the inclination of the fixed plane π to the plane ABl. The anharmonic ratio along AB is given by k; m, h, and α have the same meanings as in type III of the plane.

TYPE V.

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 0 & 0 & 1 & f \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & 0 & 0 & f \end{vmatrix}} = \frac{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 0 & 0 & 1 & f \end{vmatrix}}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & 0 & 0 & f \end{vmatrix}} + \alpha;$$

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 0 & q & -p & 1 \\ 1 & 0 & 1 & f \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & 0 & 0 & f \end{vmatrix}} = \frac{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 0 & q & -p & 1 \\ 1 & 0 & 0 & f \end{vmatrix}}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & 0 & 0 & f \end{vmatrix}} + k\alpha \frac{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 0 & 0 & 1 & f \end{vmatrix}}{\begin{vmatrix} x & y & x & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & 0 & 0 & f \end{vmatrix}} + \frac{k}{2}\alpha^2 + h\alpha;$$

$$\frac{1}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & 0 & 0 & f \end{vmatrix}} = \frac{1}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & 0 & 0 & f \end{vmatrix}} +$$

$$m\alpha \frac{\begin{vmatrix} x & y & z & 1 \\ 1 & b & c & 1 \\ a & q & -p & 1 \\ 0 & 0 & 0 & f \end{vmatrix}}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & c & 0 & f \end{vmatrix}} + \left(\frac{km}{2}\alpha^2 + n\alpha\right) \frac{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 0 & 0 & 1 & f \end{vmatrix}}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & 0 & 0 & f \end{vmatrix}} + \frac{km}{6}\alpha^3 + \frac{(hm+n)}{2}\alpha^2 + g\alpha.$$

The coordinates of the invariant point A are (a,b,c); the direction of the line Al is determined by p and q; f determines the position of the invariant plane through the line Al. k, h, and α have the same meanings as in type III of the plane. m, n and g have analogous meanings with respect to the bundle of invariant twisted cubics which constitute the path curves of every one parameter group of transformations of this type.

TYPE VI.

$$\frac{x_1-a}{z_1-c} = \frac{x-a}{z-c}; \quad \frac{y_1-b}{z_1-c} = \frac{y-b}{z-c}; \quad \frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & m & 0 & 0 \\ 0 & n & 1 & 0 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ 0 & l & 0 & 1 \\ 1 & m & 0 & 0 \\ 0 & n & 1 & 0 \end{vmatrix}} = k \frac{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & m & 0 & 0 \\ 0 & n & 1 & 0 \end{vmatrix}}{\begin{vmatrix} x & y & z & 1 \\ 0 & l & 0 & 1 \\ 1 & m & 0 & 0 \\ 0 & n & 1 & 0 \end{vmatrix}}.$$

The coordinates of the invariant point D are (a,b,c); l, m, n fix the position of the plane of invariant points. k is the anharmonic ratio along each invariant line through the point D.

TYPE VII.

$$\frac{x_1-a}{z_1-c}=\frac{x-a}{z-c};\quad \frac{y_1-b}{z_1-c}=\frac{y-b}{z-c};\quad \frac{1}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ 1 & m & 0 & 0\\ 0 & n & 1 & 0\end{vmatrix}}=\frac{1}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ 1 & m & 0 & 0\\ 0 & n & 1 & 0\end{vmatrix}}+\alpha.$$

The coordinates of the point D are (a,b,c); m and n determine the position of the plane through D every point of which is invariant. α is the characteristic constant along each line of the bundle through D.

TYPE VIII.

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_2 & b_2 & c_2 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\end{vmatrix}}=k\frac{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_2 & b_2 & c_2 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\end{vmatrix}};\quad \frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\end{vmatrix}}=k^{1-r}\frac{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\end{vmatrix}};$$

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\end{vmatrix}}=\frac{\begin{vmatrix} x & y & z & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ a_2 & b_2 & c_2 & 1\end{vmatrix}}.$$

In these equations (a,b,c) are the coordinates of some one of the invariant points of the line AB, and p and q determine the direction of this line; (a_1,b_1,c_1) and (a_2,b_2,c_2) are the coordinates of C D respectively. k is the anharmonic ratio along each of the invariant lines in the plane ABC; k^{1-r} is that along each invariant line in the plane ABD.

TYPE IX.

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ 1 & p & q & 0\\ 1 & p_1 & q_1 & 0\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & q_1 & 0\end{vmatrix}}=k\frac{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ 1 & p & q & 0\\ 1 & p_1 & q_1 & 0\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & q_1 & 0\end{vmatrix}};\quad \frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & q_1 & 0\end{vmatrix}}=k\frac{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & q_1 & 0\end{vmatrix}};$$

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\\ 1 & p_1 & q_1 & 0\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & q_1 & 0\end{vmatrix}} = \frac{\begin{vmatrix} x & y & z & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\\ 1 & p_1 & q_1 & 0\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & q_1 & 0\end{vmatrix}} + \alpha.$$

The coordinates of the invariant point A are (a,b,c) while p and q determine the direction of the line Al; (a_1,b_1,c_1) are the coordinates of some point on the line BC, every point of which is an invariant point, while p_1 and q_1 determine the direction of BC. k is the anharmonic ratio along each line of the pencil with vertex at A and lying in the plane ABC; α is the characteristic constant along the line Al.

TYPE X.

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a & p & q & 0\\ 1 & p_1 & q_1 & 0\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & q_1 & 0\end{vmatrix}} = k\frac{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ 1 & p & q & 0\\ a & p_1 & q_1 & 0\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & q_1 & 0\end{vmatrix}}; \quad \frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & x_1 & 0\end{vmatrix}} = k\frac{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p & q & 0\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & q_1 & 0\end{vmatrix}};$$

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ 1 & p & q & 0\\ 1 & p_1 & q_1 & 0\end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & q_1 & 0\end{vmatrix}} = \frac{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ 1 & p & q & 0\\ 1 & p_1 & q_1 & 0\end{vmatrix}}{\begin{vmatrix} x & y & z & 1\\ a & b & c & 1\\ a_1 & b_1 & c_1 & 1\\ 1 & p_1 & q_1 & 0\end{vmatrix}}.$$

(a,b,c) are the coordinates of some point on the line AD, and (p,q) determine the direction of AD; (a,b,c) are the coordinates of some point on the line BC, while (p,q) determines the direction of BC. k is the anharmonic ratio along every line joining a point on AD with a point on BC.

TYPE XI.

$$\dfrac{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 0 & 0 & 1 & f \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 1 & p & q & 0 \end{vmatrix}} = k \dfrac{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 0 & 0 & 1 & f \end{vmatrix}}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 1 & p & q & 0 \end{vmatrix}}; \qquad \dfrac{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 0 & q & -p & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 1 & p & q & 0 \end{vmatrix}} = \dfrac{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 0 & q & -p & 1 \end{vmatrix}}{\begin{vmatrix} x & y & x & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 1 & p & q & 0 \end{vmatrix}}.$$

$$\dfrac{1}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 1 & p & q & 0 \end{vmatrix}} = \dfrac{1}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ a_1 & b_1 & c_1 & 1 \\ 1 & p & q & 0 \end{vmatrix}} + \alpha.$$

The coordinates of the point A are (a,b,c); the direction of the line Al is determined by p and q. The coordinates of the point B are (a_1,b_1,c_1); f determines the inclination of the plane π to the plane of ABl. k is the anharmonic ratio along each of the lines of the pencil in the plane ABl passing through B; α is the characteristic constant along each line of the pencil in the plane π passing through A.

TYPE XII.

$$\dfrac{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 0 & 0 & 1 & f \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & 0 & 0 & f \end{vmatrix}} = \dfrac{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 0 & 0 & 1 & f \end{vmatrix}}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & 0 & 0 & f \end{vmatrix}}. \qquad \dfrac{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 0 & q & -p & 0 \\ 1 & 0 & 0 & f \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & 0 & 0 & f \end{vmatrix}} = \dfrac{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 0 & q & -p & 0 \\ 1 & 0 & 0 & f \end{vmatrix}}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & 0 & 0 & f \end{vmatrix}} + \alpha.$$

$$\dfrac{1}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & 0 & 0 & f \end{vmatrix}} = \dfrac{1}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & 0 & 0 & f \end{vmatrix}} + \alpha'.$$

(a,b,c) are the coordinates of some point on the line of invariant points and p and q determine the direction of this same line.

TYPE XIII.

$$\frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 0 & q_1 & p_1 & 0 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & p_1 & q_1 & 0 \end{vmatrix}} = \frac{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 0 & q_1 & p_1 & 1 \end{vmatrix}}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & p_1 & q_1 & 0 \end{vmatrix}}; \quad \frac{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p_1 & q_1 & 0 \\ 0 & q & -p & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & p_1 & q_1 & 0 \end{vmatrix}} = \frac{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p_1 & q_1 & 0 \\ 0 & q & -p & 1 \end{vmatrix}}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & p_1 & q_1 & 0 \end{vmatrix}} + \alpha.$$

$$\frac{1}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & p_1 & q_1 & 0 \end{vmatrix}} = \frac{1}{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & p_1 & q_1 & 0 \end{vmatrix}} + m\alpha \frac{\begin{vmatrix} x & y & z & 1 \\ a & b & c & 1 \\ 1 & p_1 & q_1 & 0 \\ 0 & q & -p & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ a & b & c & 1 \\ 1 & p & q & 0 \\ 1 & p_1 & q_1 & 0 \end{vmatrix}} + \frac{m}{2}\alpha^2 + h\alpha.$$

The coordinates of the point A of the invariant figure are (a, b, c); p and q determine the direction of the line Al; p_1 and q_1 determine the direction of the line AB, every point of which is an invariant point. m and h have the same meanings as k and h in type III in the plane. m and h are the same for all planes passing through Al.

If k be made equal to one in the equations of type IV, the result is equations of type XIII as above.

CANONICAL FORMS.

The canonical forms of the thirteen types of projective transformations in space may be obtained in the same way as that employed for the forms in the plane.

Type I. $x_1 = kx;\ y_1 = k^{1-r}y;\ z_1 = k^{1+r-rs}z.$

Type II. $x_1 = kx;\ y_1 = k^{1-r}y;\ z_1 = z + \alpha.$

Type III. $x_1 = kx;\ y_1 = ky + kx\alpha;\ z = z + \alpha'.$

Type IV. $x_1 = kx;\ y_1 = y + \alpha;\ z_1 = z + m\alpha y + m\frac{\alpha^2}{2} + n\alpha.$

Type V. $x_1 = x + \alpha;\ y_1 = y + k\alpha x + k\frac{\alpha^2}{2} + h\alpha;\ z_1 = z + m\alpha x +$

$$\left(km\frac{\alpha^2}{2} + n\alpha\right)y + km\frac{\alpha^3}{6} + (hm + n)\frac{\alpha^2}{2} + g\alpha.$$

Type VI. $x_1 = x;\ y_1 = y;\ z_1 = kz.$

Type VII. $x_1 = x;\ y_1 = y;\ z_1 = z + a.$

Type VIII. $x_1 = kx;\ y_1 = k^{1-r}y;\ z_1 = z.$

Type IX. $x_1 = kx;\ y_1 = ky;\ z_1 = z + a.$

Type X. $x_1 = kx;\ y_1 = ky;\ z_1 = z.$

Type XI. $x_1 = kx;\ y_1 = y;\ z_1 = z + a.$

Type XII. $x_1 = x;\ y_1 = y + a;\ z_1 = z + a'.$

Type XIII. $x_1 = x;\ y_1 = y + a;\ z_1 = z + may + m\dfrac{a^2}{2} + na.$

On the Skull of Xerobates (?) undata Cope.

Contributions from the Paleontological Laboratory, No. 32.

BY J. Z. GILBERT.

In the University of Kansas Museum there are a number of specimens of turtles collected from the Loup Fork Beds of Phillips Co, Kansas, by Messrs. Sternberg, West and Overton. This material has been entrusted to me by Dr. Williston, under whose advice and direction I have thoroughly studied it, with the result that two well-defined species have been made out, the detailed description of which will be given in a later paper. A well-preserved and nearly complete skull of one of the species is of so much importance that it has been thought worth while to give here a preliminary description of it in advance of the fuller paper,

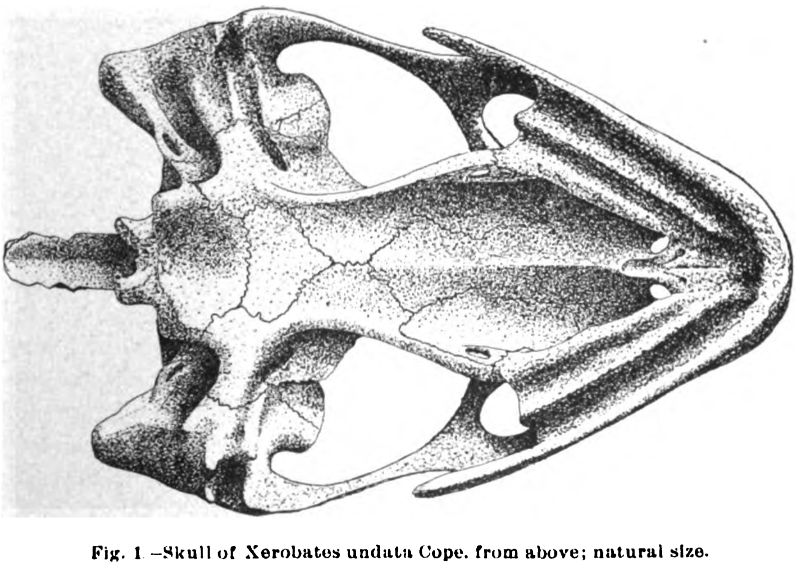


Fig. 1.—Skull of Xerobates undata Cope, from above; natural size.

inasmuch as this part of the anatomy of these Miocene reptiles has hitherto been wholly unknown. It is provisionally referred for the present to *Xerobates* (*Testudo*) *undata* Cope, but its specific identity is more or less doubtful, inasmuch as the original description of the species to which it is referred is very incomplete and imperfect.

The skull is rounded in the premaxillary region, and is rather narrow and long. The outline of the base of the skull may be described as follows: The sides in front of the anterior margin of the infratemporal fossæ make an angle of sixty degrees with each other; from this same margin to the quadrate an angle of twelve degrees. Seen from the side, the skull thins posteriorly, the dorsal and ventral planes making an angle of eight degrees with each other. The dorsal plane lies upon the highest portion of the supraoccipital crest, and the upper, flattened surface of the skull between the orbits; the ventral or basal plane extending from the lower margin of the outer maxillary cutting edge through the quadrates. Between the two points touched by the dorsal plane there is a long, shallow concavity, which merges into the broad, shallow depression in the region of the fronto-parietal suture. The supraoccipital crest is small, and arches only a little above the otherwise gently downward sloping bone.

The anterior nares are one-third wider than high; they are large and quadrilateral. From the highest portion of the cranium the face slopes downward and outward, with a small degree of convexity. The orbits are large and deep, round in outline, and look obliquely outward, forward and upward. The sutural union between the frontal and postfrontal occurs immediately above the middle of the orbit. The postorbital and infraorbital bars are thin and plate-like. The skull throughout, in fact, is characterized by its general lightness of bone. The supratemporal fosæ are large, oval, with their long diameter making an angle of forty-five degrees with the sagittal plane. They look obliquely backward, upward and slightly outward. On the posterior border of these fossæ there is a prominent, quadrilateral, short, stout process for muscular attachment. This process is concave on its upper and anterior surface, and its long axis stands obliquely inward and forward. It is formed by the squamosal and prootic. Below the temporal bar there is a broad, deep notch, the plane of which looks immediately outward, with only a slight upward and forward obliquity. The process from the maxilla extends prominently backward for about 12 mm.

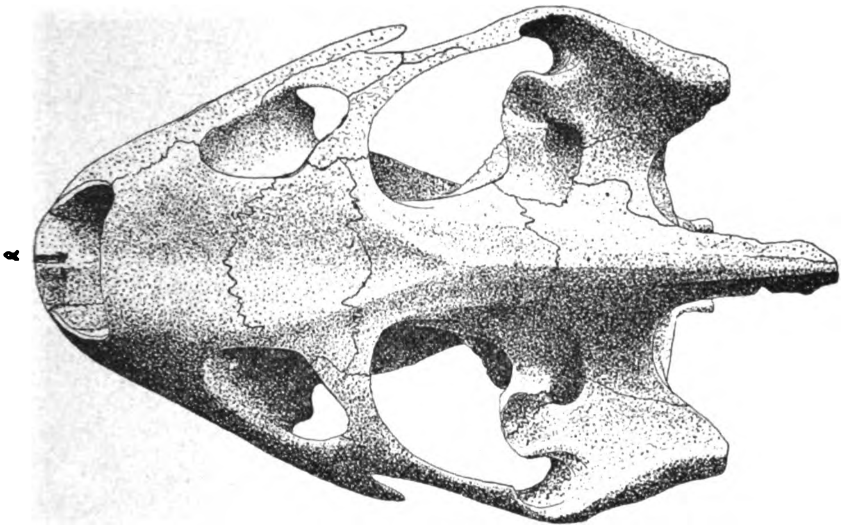


Fig. 2.--Skull of Xerobates undata Cope, from below; natural size.

The premaxillaries are sharp at their lower border, which is gently concave in side view. Seen from below the lower plane is very uneven, with the basisphenoid only flattened.

The premaxillaries are straight on the edge below and arched to a point above. They form almost the entire floor of the anterior nares. They are convex transversely in the middle, and doubly convex in front. They have no articulation with the palatine; in front below there is a deep, round fossa for the reception of the beak of the mandible. The posterior nares at the posterior margins of the bones are rounded. The premaxillaries articulate posteriorly with the stout descending process of the vomer. The maxillaries have two cutting edges, the inner one with its plane much above the plane of the outer. They send up a broad process, thinned and narrowed above to join the prefrontal. The outer cutting edges increase in thickness from a sharp, serrate one, to one four millimeters thick above. The pterygoids, palatines and vomers together form a deep ascending channel, broadest a little in front of the palatine foramina; the channel is divided by a low ridge in the middle, which in the anterior part of the vomer is thin and sharp and curves downward. The posterior process extends from the outer cutting edge, instead of from the second, as in many turtles. It is thin, acutely angled, and extends slightly outward and downward below the basal plane, while the outer surface slopes at an angle of forty-five degrees. The grove on the inner

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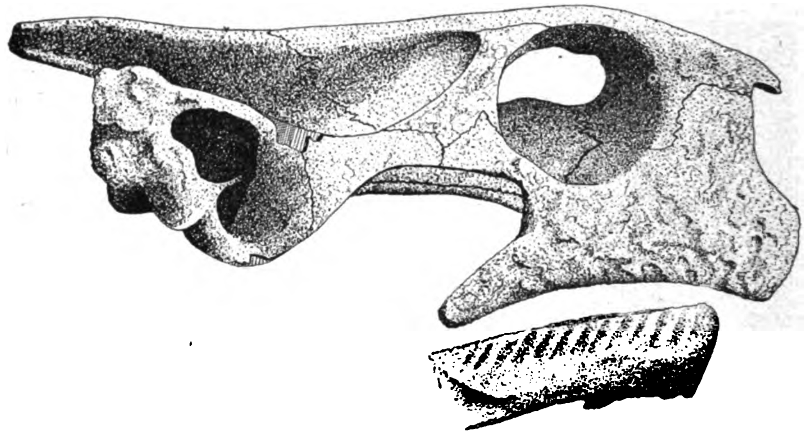

Fig. 3.—Skull of Xerobates undata Cope, natural size.

side also deep. The maxilla does not curve inward anteriorly to meet the premaxilla, but is rounded in this region by thinning to a sharp edge. The inner edge of the maxilla makes a low, almost serrate border, becoming lower anteriorly until it merges into the level bottom of the inner channel.

The palatine foramen is situated near the sutural union of the palatine and pterygoid. There is a strong ridge below the foramen and, on either side, a narrow, shallow grove. The pterygoids have the anterolateral sides projecting as long, narrow, rod-like processes, standing at an angle of thirty-three degrees with each other. They unite for a short distance between the basisphenoid and vomer deepening anteriorly the palatine region. The lateral edges approach each other to just behind the anterior end of the basisphenoid, where they diverge gradually, becoming less prominent, and finally terminating in the postero-lateral process. The vomer has very irregular margins and a medial ridge throughout its entire length, except at the anterior end, where a small, deep, narrow, U-shaped groove occurs. Anteriorly the vomer sends down a strong, triangular column, the anterior surface of which is deeply and angularly channelled. The jugal is an hourglass-shaped bone, and is very light. The quadrato-jugal is a much wider bone than the jugal. It is broad, thin and plate like, expanding anteriorly to a width of twelve millimeters, posteriorly to eight.

The deep external tympanic fossa of the quadrate is oval or inverted reniform, with its long axis directed downward and forward. The squamosal is arched over the tympanic fossa from the

inner, almost vertical surface, and takes no part in the formation of a false roof.

The occipital condyle is triangular, broadest above, with a slight depression on the posterior surface. Back of the basisphenoid there is a well-marked concavity, the anterior and lateral margins of which coincide with those of the bone itself. The basioccipital processes are strong.

The exoccipital fossa is a shallow, round pit, with its ventral wall quite low. Immediately above this concavity, and over the sutural union betweeen the exoccipital and the opisthotic, there is a long, shallow concavity.

The epiotic is only partially fused with the supraoccipital. The external carotid foramen is midway between the tympanic rim and the zygomatic arch; there is a shallow crease curving upward and forward from this foramen. On the posterior margin of the temporal fossa and in front and exterior to the carotid foramen there is a large, stout, dorsally concave tuberosity, the suture between the prootic and the squamosal passing through its middle and through the carotid foramen. A broad, shallow groove separates the tuberosity from the zygoma, and there is another on the inner side. The external auditory meatus is oval; it looks downwards, backwards and slightly outwards.

The basisphenoid is triangular, with its base posterior; the surface is in a plane of about twenty-six degrees with that of the base of the skull.

The parietals form no portion of a false roof; they are rounded above, and there is a perceptible ridge arising from each antero-lateral process and fusing with its mate a little in front of the occipital crest. The antero-lateral margin is at an angle of forty-five degrees with the horizontal. There is a broad, shallow depression on the upper surface.

The frontals are much wider than long, with a broad, median depression, a continuation of that from the parietals. The antero-lateral ridges of the parietals continue on the frontals. The rhinencephalic groove below is not bridged over.

The prefrontals are strongly convex forward and laterally. The compressed top and flattened sides give to this region a decidedly quadrilateral shape. The anterior margin is concave antero-posteriorly and convex vertically. On the inner side and from the posterior part, a strong triangular process extends inward and backward to meet the upward and forward process of the vomer. The anterior ventral part of this process has an angular ridge,

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Fig. 4.—Mandible of Xerobates undata Cope, from above; natural size.

which, with its mate, is so prominent that it constricts the nasal cavity at this place, forming a smaller, secondary nasal cavity behind. This lateral is equitrilateral in cross-section, while the anterior portion is distinctly quadrilateral.

The mandibles have two strong cutting edges enclosing a deep grove between them, the channel for the inner ridge of the maxilla. The symphysial portion slopes forward at an angle of fifteen degrees, and the lower portion of the symphysial region extends backward so as to cause the otherwise angular symphysis to be broadly concave.

A Plan for Increasing the Capacity of the Steam Heating Plant of the Spooner Library, University of Kansas.

BY FRANK E. WARD.

The buildings of the University of Kansas are all heated from one boiler-house, which is on the south side of the hill and 400 feet from Snow Hall and 500 feet from Fraser Hall and the physics building which are on the top of the hill. The Spooner library building is placed just below the top of the hill on the northeast side, and 1300 feet from the boiler house.

It is not the purpose of the writer to offer criticism or suggestions regarding the method of heating the library building; but to show how a difficulty in forcing the steam over to the library was overcome by a simple plan, which has now been in successful operation for two years.

After the steam-heating surfaces have been carefully calculated and the pipes put in place in a large building like the library, it is often found that unforseen drafts and exposures necessitate changes or enlargement of surfaces, and at best the plant is often insufficient in very cold weather. Several changes were made in this case until the heat was very well distributed. But when severe weather set in it was found impossible to heat the library and it was closed several times on this account, while the other buildings which are heated from the same source were almost too warm.

The low pressure gravity system used in heating all the University buildings, except the physics building, in which all the condensation returns to the boilers by gravity, is as follows: From a battery of four boilers there are two pipes which carry away steam and return the condensation. These pipes maintain an equal pressure which varies, but in this case does not exceed twenty pounds per square inch. From these main pipes, branches are taken off which lead to and return from all the above mentioned buildings. Fig. 1 represents the 10-inch main and 6-inch return.

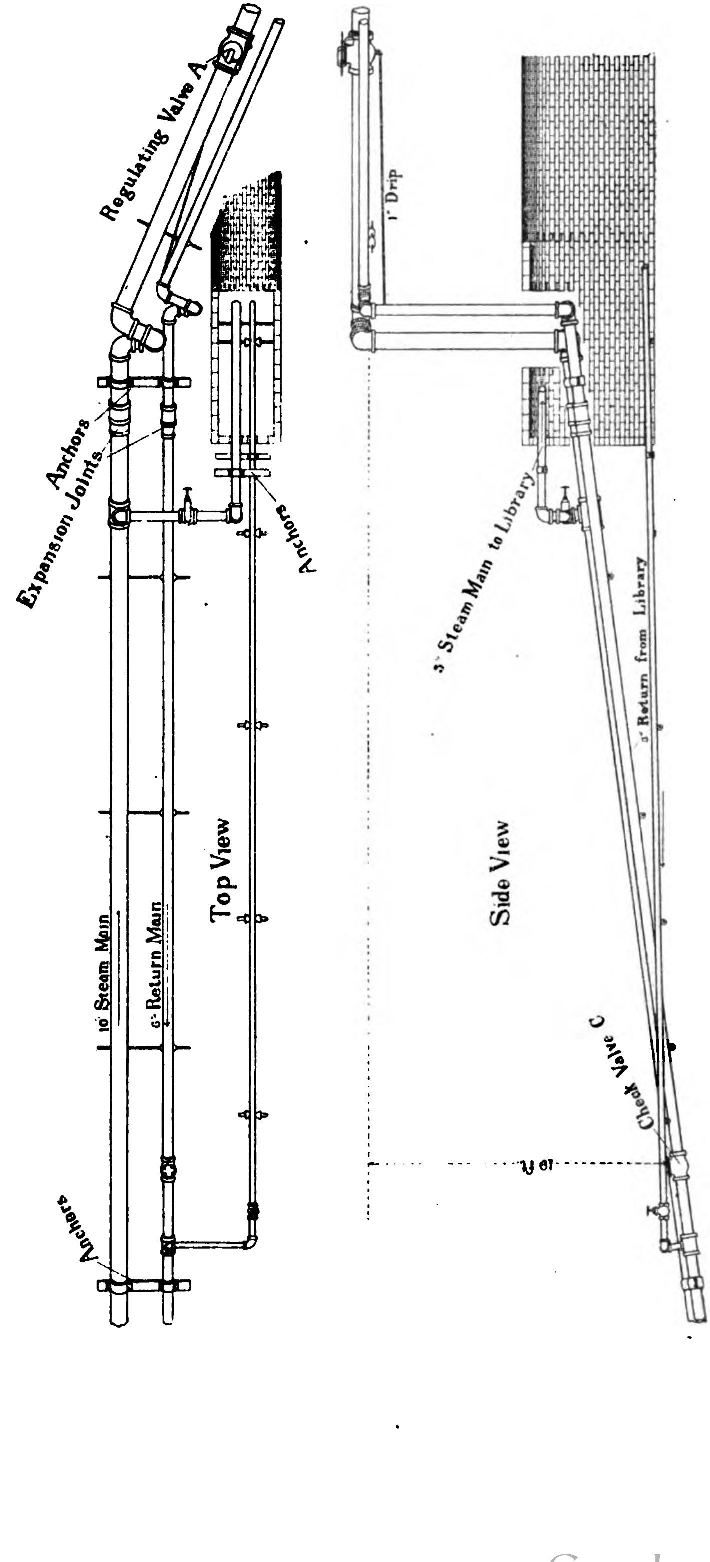
Regulating Valve A
Anchors
Expansion Joints
Anchors
10" Steam Main
6" Return Main
Top View
Anchors
1" Drip
3" Steam Main to Library
Return from Library
Side View
Check Valve C
19 ft

Fig. 1.

These pipes connect low enough to run through a tunnel nearly 1000 feet long to and from the library. The steam pipe starts into the top of the tunnel and gradually descends to the middle at the far end where the return pipe starts and descends to the bottom at the end of the tunnel as shown.

After a trial of two winters, it was found necessary to maintain a greater pressure in these pipes than in the mains leading to and returning from the buildings which are nearer and on a higher level, the natural draft being upward.

Propositions were made by reliable firms, which would have reached the required end, but they were expensive both in first cost and in operation.

The writer's plan which was put into operation is:

First—a regulating valve, Fig. 2, placed in the 10-inch main at A (see Fig. 1) to form a back pressure at that point to any desired amount (say 5 pounds) which can be regulated by the weight W.

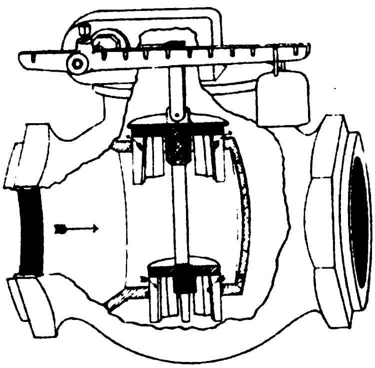

Fig. 2.

Thus the back pressure will fill the long pipes leading to the library with steam at a pressure of 5 pounds or any desired amount to overcome the difference between its heating capacity and that of the remaining buildings. Then as the pressure rises it is permitted to pass through the regulating valve so that when the back pressure is 10 pounds, the pressure beyond the valve is 5 pounds. If this should heat the library too much the weight can be placed back a notch and the back pressure reduced by allowing the boiler pressure to reduce, and the temperature of the other buildings will not be changed. These valves are used successfully in many places similar to this. In our case, however, the return main will have an equal pressure effected by the necessary connection to the

steam main, which will cause the condensation of water and moist vapors to back up into the buildings before the regulating valve opens. This is usually corrected by the use of pumps or traps which hold the condensation under control, and it was proposed to put in an expensive plant of this kind until the present plan was suggested.

Second—a check valve, Fig. 3, is placed in the 6-inch return main at C (Fig. 1) in such a way as to prevent a back pressure of 10 pounds from passing up into the return pipes which have a

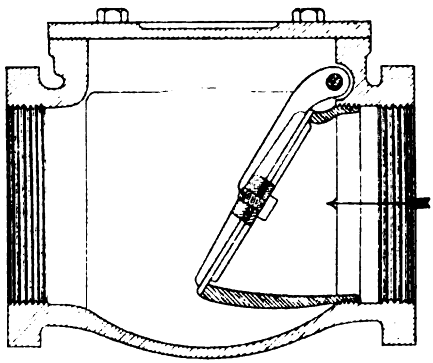

Fig. 3.

pressure of 5 pounds. Then when the water and moist steam gather on the low pressure side of the check valve, they separate and the water rises until the column thus formed increases the pressure enough to open the check valve C, the water running through until the valve is closed again by the pressure on the other side. This valve is entirely self-acting, and the amount of water that is allowed to pass out at one time is very small, for when the valve is open the area on both sides is equal and when the water runs through enough to make the pressures both equal the valve will close, but when closed the area of the top is greater than that of the bottom and the amount of water in addition which is required to equalize the pressure is all that passes through at one time. This makes the operation frequent and keeps the water in the boilers at about the same level. At night when the steam goes down there is no pressure to hold up the water, so the pipes empty themselves, and freezing is prevented. The extra water is needed in the morning, so that in every respect the result is satisfactory.

On special occasions when Library Hall only is heated the weight W is placed out on the end of the arm and the other buildings will not be heated. When heat is not wanted in the library and is needed elsewhere the weight is removed entirely. This requires but little time and is all that is done to operate both valves and equalize the heat.

When the weather is not too severe the boilers are capable of supplying the demands, but they are taxed to their utmost when the wind is very cold.

The Hyperbolic Spiral—Its Properties and Uses.

BY WALTER K. PALMER.

There are many well known and useful varieties of spirals. Of these, one of the most interesting in its properties and uses is the Hyperbolic Spiral.

It is called *Hyperbolic* because its equation is of just the same form as the rectangular equation of the common hyperbola, and because it may be plotted by transforming the rectangular co-ordinates of this hyperbola into polar co-ordinates. It is also called the "*Reciprocal*" or "*Inverse*" *Spiral*, from the fact that one co-ordinate varies inversely as the other, or equals a constant times the reciprocal of the other. These facts are shown fully by the general form of the curve's equation, which is $r = \frac{c}{\theta}$, where r is one co-ordinate, the radius vector, θ the other, as shown in Fig. 1, and c, a numerical constant, the magnitude of which determines the size of the particular spiral in question.

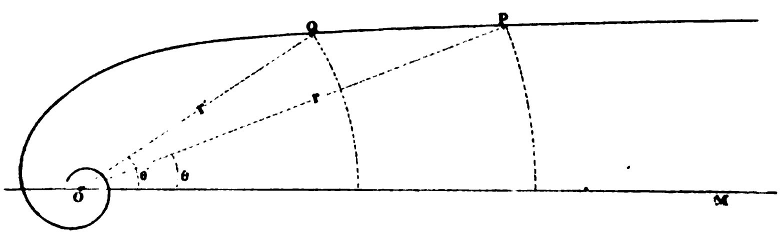

Fig. 1.

Now this relation between r and θ and c is the same at every point along the curve as well as at P. That is, at any other point Q, the particular length of r there, (r′) equals the number c, divided by the particular angular value of θ (θ') corresponding to r′.

Bearing this in mind a practical drawing board construction for the spiral may be derived readily and many interesting properties discovered.

PLOTTING THE SPIRAL.

BY TRANSFORMING THE COMMON HYPERBOLA.

The first method which suggests itself from a consideration of the characteristics of the spiral, is that of replotting a common hyperbola to polar co-ordinates. If we take an accurate plotting of one branch of an hyperbola, which can readily be made by the well known rule, and draw a number of verticals, equally spaced, as in Fig. 2, and then lay out the series of equally spaced radial lines shown in Fig. 3, a Hyperbolic Spiral may be plotted upon the latter, using Fig. 2 as an auxiliary diagram.

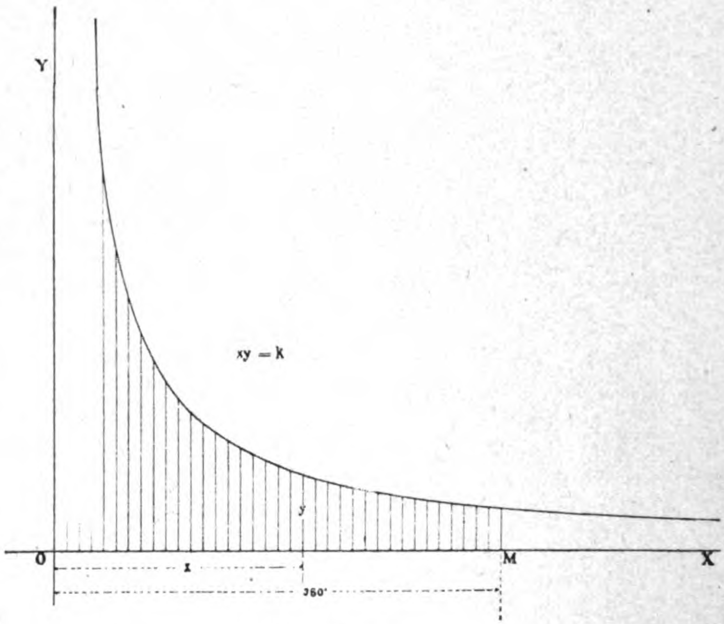

Fig. 2.

Commencing, say, with the longest vertical of Fig. 2, lay it off with the dividers upon the radial line No. 3, and the next on No. 4, and so on. The points thus marked off are points on the line of the hyperbolic spiral and may now be joined by a smooth curve.

It is plain from the characteristics of this hyperbola, as well as from an inspection of the equation of the spiral, that the curve may be continued indefinitely in either direction, never reaching either the pole, O, or the initial line.

This gives a correct construction for the spiral, but it is necessary to plan the diagram of Fig. 2 in a correct way in order to se-

cure a desired value of the constant of the spiral and to have OL the initial line of the spiral, as it should be. It is always desirable to obtain a spiral with a certain definite value for the constant, since this constant fixes the size of the spiral.

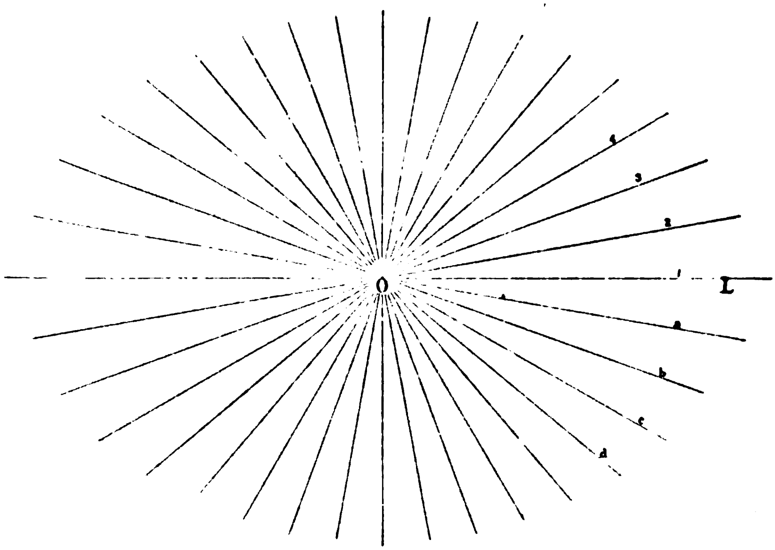

Fig. 3.

This can be attained, for the case of θ measured in degrees and r in inches, by choosing a suitable space along the horizontal line of the drawing, as OM, to represent 360 (degrees) and making the constant k of the hyperbola equal to the desired c of the spiral. Then by dividing the space OM into as many equal parts as the 360 degrees of Fig. 3 and drawing a vertical to the hyperbola at each division point, each radius vector of Fig. 3 will have a vertical corresponding to it on Fig. 2, from which its true length can at once be set off by the dividers.

A serious practical difficulty is found, however, in trying to set off accurately the lengths of the long verticals approaching OY in Fig. 2, for a very slight error to the right or left in drawing one of these long ordinates to the hyperbola produces an error many times as great in the length, which results in a serious irregularity in the spiral. This method has given satisfactory plottings, using the greatest care, but necessitates extreme care and the most skillful use of the drawing instruments.

BY CALCULATING THE RADII.

Apparently the simplest and most natural way of constructing this spiral would be to merely figure by simple arithmetic the

length of each radius vector and measure it off by use of the decimal scale, estimating tenths of tenths on the scale, thus setting off results to hundredths. And this is entirely satisfactory in the results attained, but it is found tedious figuring each radius and measuring it off. A diagram from which each radius line can be set off at once by the dividers is desirable. Then a few of the radii may be measured as a check on the graphical work.

Hence the following has been devised as by far the best drawing board construction for this spiral. It obviates all the difficulties of of the preceeding and is very satisfactory in all respects.

PRACTICAL DRAWING BOARD CONSTRUCTION FOR THE SPIRAL.

Lay out the diagram of Fig. 4, drawing first the two lines OY and OX at right angles. Then choose the distance OM, as large as the size of the paper will permit, allowing room at the right of M, for at least ½ OM. OM is to represent the degrees of one circumference, 360. Then draw AH parallel to OM at a distance above, equal to the constant c, of the equation $r=\frac{c}{\theta}$, in the same scale as the 360. That is, if we choose 120 for the value of the constant c, and this gives a very satisfactory size of the spiral, c will be made $\frac{120}{360}=\frac{1}{3}$ of the length chosen for OM. Having drawn AH, draw AD at unit's distance to the right of OY to the same scale which is to be used for r on the plotting. If r is to be measured in inches, as is usually the case, make AD one inch to the right of OY.

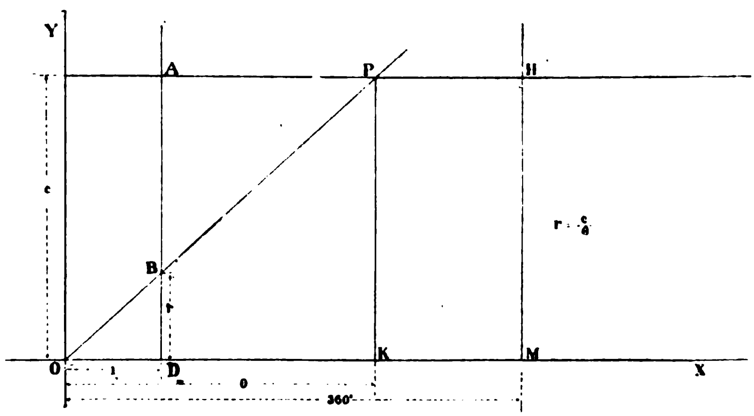

Fig. 4.

Now if any value of θ be measured off in degrees to the scale of OM, that is as the correct fractional part of OM (as OK), the corresponding value of r in the equation $r = \frac{c}{\theta}$ is at once seen at DB: for, drawing KP, and PO cutting AD at B, we have by similar triangles $\frac{BD}{DO} = \frac{KP}{KO}$, or $r = \frac{c}{\theta}$.

To utilize this prepare the diagram of radial lines for the plotting, as in Fig. 3, using as many lines as possible. It is well to divide each quadrant into eighteen parts by trisecting the quadrant, then trisecting each part by the dividers and then bisecting each of these parts. Then each space will represent five degrees. For the sake of accuracy these divisions should be performed on as large a circle as can be drawn about the pole O on the sheet of paper used.

Now divide YH, of the auxiliary diagram, Fig. 4, into the same number of parts (72), continuing spaces of the same size to the right of H. Then, beginning at $\theta = 360$, i. e., at H, draw the line to O, determining the length of r, and with the dividers accurately set off this length on the corresponding radial line of the plotting, which is OL. Use the points to the right and left of H as far as possible, points to the left of H giving constantly increasing values of r and being laid off on the plotting in the reverse order of the numbers, i. e., OL, Oa, Ob, etc. In the case of values of YP less than YA, r will be found on BA produced upward.

For accuracy's sake, when drawing the radials to O, a small circle, say $\frac{1}{16}''$ in diameter, should be drawn about the point O, and all lines stopped at the circle to prevent obscuring the point and thus to make it possible to draw each line exactly radial with respect to O. This suggestion should be observed in laying out the radial lines of the plotting also.

When all the values of r which can be secured from the diagram have been transferred to the plotting there will be a series of points which determine the desired spiral. Through these draw a smooth curve and the result will be the hyperbolic spiral of the predetermined dimensions. Some of the radii should now be checked by calculation, and should be correct to .01".

This construction can be much facilitated by using co-ordinate paper, ruled in inches and tenths, for the diagram of Fig. 4. OM can then be chosen so that an exact number of the smaller spaces represent a degree, when no drawing will be required beyond locating YH. A straight edge can then be set at once to any desired

value of the angle along YH, and the corresponding radius immediately maked off on AD, or read off directly in inches and decimals as desired.

PROPERTIES OF THE HYPERBOLIC SPIRAL.

MATHEMATICAL PROPERTIES.

This spiral is found to possess many interesting properties. If a spiral of any chosen size be cut from a sheet of uniform thin metal, pearwood, hard rubber, celluloid or any other suitable material, so that by its use this particular size of spiral can be easily and quickly traced upon a drawing in any desired position, a number of interesting mathematical operations may be performed graphically. Among these the most notable are the trisection or multisection of any angle, the finding of reciprocals of numbers, the laying out of the regular polygons and the rectification of arcs.

Graphical Operations.

THE SPIRAL INSTRUMENT.

For convenience, choose a spiral with the constant term made equal to 100. Fig. 5 shows the outlines of an instrument constructed as suggested, with a spiral whose equation is $r=\frac{100}{\theta}$ for the working curve. The cut is just half the actual size of the instrument.

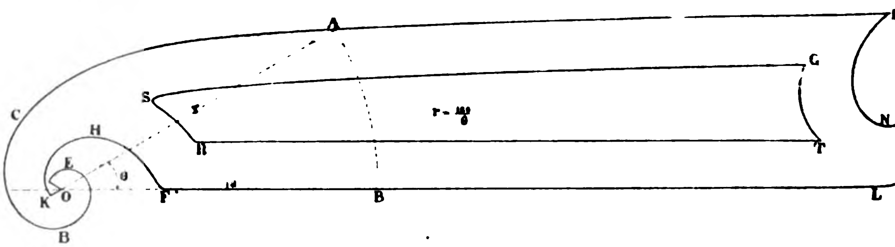

Fig. 5.

The curve EBCD, Fig. 5, is the spiral, O is the pole and OL its initial line. The curve FHK and the V-shape with vertex exactly at the pole are to permit the pole being placed exactly at a point on the drawing, and the construction is such that at the same time the initial line, OL, can be brought, readily, into co-incidence with any straight line through O. The space bounded by the curves TRSG, is cut out simply to lighten the instrument. And the curves DNG and QWL are merely to give a "finish" to it.

Preferably this curve should be made from transparent celluloid, about $\frac{3}{64}$" thick. The transparency is very helpful in permitting accurate setting of the instrument.

RECIPROCALS.

From the character of the spiral outline of the instrument, we have the fact that any radius of the spiral, r, Fig. 5, is equal to 100 times the reciprocal of the angle in degrees which the radius, r, makes with the initial line OL. So if we wish the reciprocal of any number, as, say, 119, it is readily found by setting off 119 by means of an ordinary protractor, upward from a horizontal line, as shown in Fig. 6, and applying the spiral instrument.

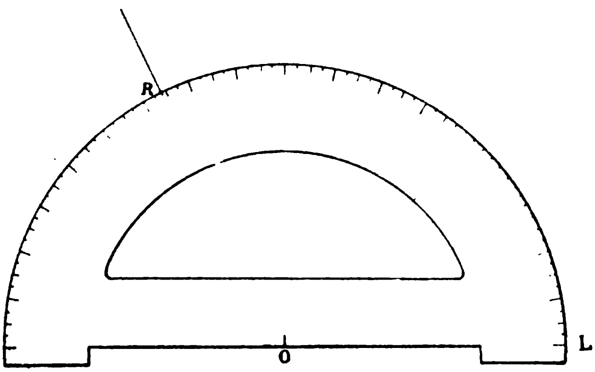

Fig. 6.

Fig. 7 shows the angle laid out by the protractor. Upon this angle, with the pole at the vertex of the angle and the initial line co-incident with OL, apply the instrument as shown in the figure.

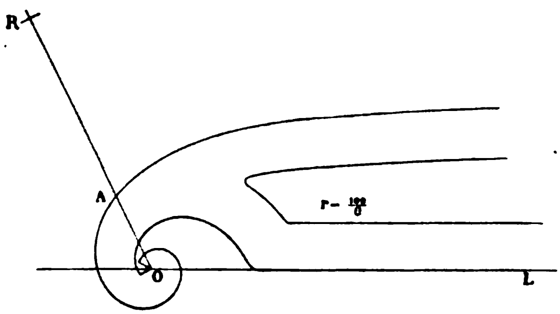

Fig. 7.

Mark with the lead pencil along the outline of the spiral where it crosses OR. Then measure accurately, with decimal scale, the distance from O to this intersecting point, A. Then this measure-

ment OP is 100 times the reciprocal of 119, and it only remains to move the decimal point two places to the left, when the result is the desired reciprocal of 119. Thus the reciprocal of any number within the physical limits of the particular instrument may be found. Mathematically there is no limit, as the spiral makes an infinite number of turns about the pole and extends an infinite distance outward.

MULTISECTION OF ANGLES.

First Method.—1. To Bisect an Angle: Take any angle, LOD, Fig. 8. Place the instrument as before, initial line coinciding with OL, pole at vertex of angle. Strike a spiral arc at A, across OD. Draw arc AB and set off BC=OB from point B. Draw the arc EC, connect E to O, and OE bisects angle BOA. For, by the properties of the spiral, $\angle$ COE=½ $\angle$ BOA, since the radius OE was made =2 OA, the angles being inversely as the radii.

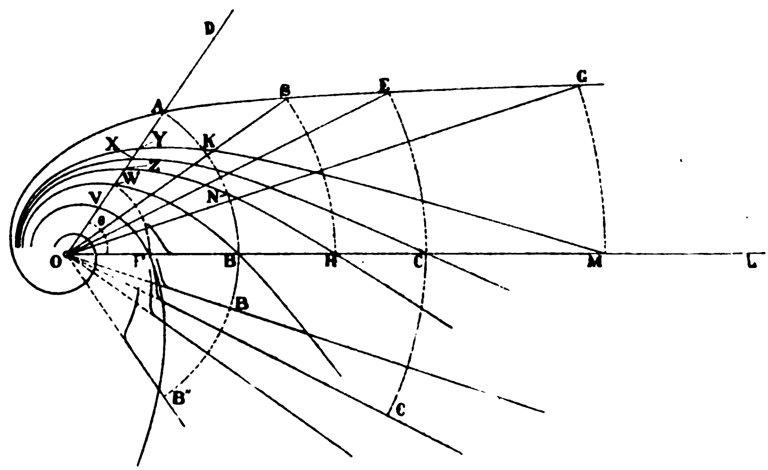

Fig. 8.

2. To Trisect an Angle: Similarly, if it is desired to trisect the angle, set off OB three times and MOG=⅓ BOA. Or if any fractional part be desired, as the one-fifth or one-seventh, set off this "primary radius," OA, a number of times equal the denominator of the fraction, as five or seven.

3. In General: For any fractional part of the angle, set off OA, or OB, a number of times equal to the fraction inverted.

Second Method.—Better than this, the same result may be reached as follows: Take, for illustration, the case of trisection again. As before, draw the arc AB, Fig. 8, and set off the primary radius, OB, three times. Then turn the instrument till the spiral passes

through M, and its intersection K, with the arc AB determines the third of $\angle$ LOD. For as before, $\angle$ MOG$=\frac{1}{3}$ $\angle$ LOD, and as every point of the instrument of course moves through the same angle when the instrument is turned, point A moves through one-third $\angle$ LOD from A to K.

Relations Between Fractions of Angle and Intercepts of Sides.—The turning of the instrument thus suggests a series of interesting results which may be developed by setting in this way to distances of different numbers of times the primary radius, first along OL and then along OD, with the instrument in the position shown, and striking spiral arcs across the first arc and also across other arcs at chosen distances from the pole. There are four sets of these relations:

I. Between the intercepts on the two sides of the angle. II. Between fractional part of angle and length on OL, using arc AB. Already noticed. III. Between fractional part of angle and length on OD, using arc AB. IV. Between either the intercept on OL or that on OD aud fractional part of angle, when using one or more arcs other than AB, of assumed radii.

I. In Fig. 8, turn the instrument as before, bringing the point at A down to B. Then point W bisect OA, for $\angle$B″OW$=2\angle$B″OB. $\therefore$ OW$=\frac{1}{2}$ OB$=\frac{1}{2}$ OA. If now we turn till the curve goes through F, the point V, found as W was, gives OV$=\frac{1}{3}$ OA. Continuing this, we have the series or fractional parts of the primary radius,

on OD: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, etc., etc.,

corresponding to the series:

1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, etc., etc.,

of OB along the side OL.

Instead of drawing the circular arcs each time, the instrument may be inverted, when the lengths OW, OV, etc., will be laid off alternately on the one side and the other of the angle.

Continuing to settings greater than OB, when the instrument is set at C, where OC$=2$ OB, we have OX$=\frac{2}{3}$ QA. Likewise setting to OM$=3$ OB we have OY$=\frac{3}{4}$ OA. For total angle C″OX$=\frac{2}{3}$ COA. Hence OX$=\frac{2}{3}$ OA. And total angle M″OY$=\frac{3}{4}$ MOX, so OY$=\frac{3}{4}$ OA. So we have for

4, 5, 6, 7, 8, 9, etc., times OB

$\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$, etc, of OA.

And hence, in perfectly general terms, setting to $\left(\frac{a}{n}\right)$ times the

primary radius, along OL, gives the $\left(\frac{a}{a+n}\right)$th part of OA. Or conversely, if setting to the $\left(\frac{a}{n}\right)$th part of OA. the $\left(\frac{a}{n-a}\right)$th part of OB will be determined on OL. And for the $\left(\frac{a}{n}\right)$th part of AB, along OL, the ratio of intercepts is $\left(\frac{n}{a+n}\right)$. For the $\left(\frac{a}{n}\right)$th part along OD, the ratio is $\left(\frac{n-a}{n}\right)$. And for a desired ratio, $\left(\frac{a}{n}\right)$, the intercept on OD must be $\left(\frac{n-a}{n}\right)$, and that on OL, $\left(\frac{n-a}{a}\right)$ times the primary radius.

Or, otherwise, calling the intercept on OL, x, and the corresponding intercept on OD, y, we have from the preceding the equation

$$y=\frac{x}{1+x}$$

relating the two intercepts, for any setting, and any angle, whers x and y are expressed in terms of the primary radius. Or we have

$$y=\frac{Rx}{R+x}$$

when x and y are the actual length of the intercepts, and R is the length of the primary radius.

Application: So if we should have an equation of the form

$$y=\frac{ax}{a+x},$$

where a is any constant term it can be solved at once, as follows: Set the instrument to any straight line, as OL, Fig. 8, marking the pole and striking a spiral arc. With a radius =a inches, strike a circle arc, thus determining OD for this case. Then take the value of x to be substituted in the equation, and lay it off from O, along OL, and set the instrument to it. Then the intercept on OD is the value of y sought.

Reversing the equation we have

$$x=\frac{Ry}{R-y}, \text{ and } \frac{y}{x}=\sqrt{\frac{R-y}{R+y}}=\frac{R}{R+x}=\frac{R-y}{R}.$$

II. The general relation for this case has already been noticed. *For Trisection:* If after finding W, Fig. 8, by setting to B, we make BH=OW, and strike a spiral arc on AB, determining N, then ON is again a trisector of $\angle$ BOA. For OH=$\frac{3}{2}$ of the primary radius, OA. Hence $\angle$ HOS=$\frac{2}{3}$ $\angle$ BOA, so when S moves to H, through $\angle$ $\frac{2}{3}$ BOA, A moves through $\frac{2}{3}$ BOA, to N, making $\angle$ BON=$\frac{1}{3}$ $\angle$ BOA. If S be joined to O the other trisector is drawn, making four ways in which an angle may be trisected by means of this instrument.

And so, in general terms, since either the $\left(\frac{a}{n}\right)$ or the $\left(\frac{n-a}{n}\right)$th part gives the desired $\left(\frac{a}{n}\right)$th part of the angle on the drawing, either $\left(\frac{n}{a}\right)$ or $\left(\frac{n}{n-a}\right)$ times the primary radius may be used along OL to determine it.

III, With the curve in the position YKM, at the point of trisection, K, the radius OY was equal $\frac{3}{4}$ OA, or YA=$\frac{1}{4}$ OA; for, then, total angle B′OA=B′ OK+KOA=$\theta+\frac{1}{3}\ \theta=\frac{4}{3}\ \theta$. $\therefore$ The corresponding radius vector OY=$\frac{3}{4}$ the radius for θ, =$\frac{3}{4}$ OA:

For bisection YA becomes $\frac{1}{3}$ OA, in the same way. Continuing, if we make YA=$\frac{1}{5}$ OA, the $\frac{1}{4}$ of the angle is determined, and so on. So that we have in terms of the primary radius and θ the following table of relations:

(a) Fraction of OA, measured from A, YA.	(b) Fraction of angle measured downward from A. $\angle$ AOK.	(c) Intercept on other side of angle, OM.	(d) Ratio of intercepts, (c) to (a).
$\frac{1}{2}$	I	I	2
$\frac{1}{3}$	$\frac{1}{2}$	2	6
$\frac{1}{4}$	$\frac{1}{3}$	3	12
$\frac{1}{5}$	$\frac{1}{4}$	4	20
$\frac{1}{6}$	$\frac{1}{5}$	5	30
etc.	etc.	etc.	etc.
$\frac{2}{5}$	$\frac{2}{3}$	$\frac{3}{2}$	$\frac{15}{4}$
$\frac{2}{7}$	$\frac{2}{5}$	$\frac{5}{2}$	$\frac{35}{4}$
$\frac{3}{7}$	$\frac{3}{4}$	$\frac{4}{3}$	$\frac{28}{9}$
etc.	etc.	etc.	etc.
$\frac{3}{5}$	$\frac{3}{2}$	$\frac{2}{3}$	$\frac{10}{9}$
$\frac{4}{5}$	4	$\frac{1}{4}$	$\frac{5}{16}$
$\frac{4}{7}$	$\frac{4}{3}$	$\frac{3}{4}$	$\frac{21}{16}$
$\frac{5}{7}$	$\frac{5}{2}$	$\frac{2}{5}$	$\frac{14}{25}$
$\frac{6}{7}$	6	$\frac{1}{6}$	$\frac{7}{36}$

For which, in general terms:

* $\frac{a}{n}$	$\frac{a}{(n-a)}$	$\frac{(n-a)}{a}$	$\frac{n(n-a)}{a^2}$
$\frac{a}{(n+a)}$	* $\frac{a}{n}$	$\frac{n}{a}$	$\frac{n(n+a)}{a^2}$
$\frac{n}{(n+a)}$	$\frac{n}{a}$	* $\frac{a}{n}$	$\frac{a(n+a)}{n^2}$

If, now, the fractional part of OA be measured outward from O, and the angle measured upward from the initial line, as is the customary way, instead of as assumed, the table becomes:

* $\frac{a}{n}$	$\frac{(2a-n)}{a}$	$\frac{a}{(n-a)}$	$\frac{n}{(n-a)}$
$\frac{n}{(2n-a)}$	* $\frac{a}{n}$	$\frac{n}{(n-a)}$	$\frac{(2n-a)}{(n-a)}$
$\frac{a}{(a+n)}$	$\frac{(a-n)}{a}$	* $\frac{a}{n}$	$\frac{(a+n)}{n}$
$\frac{(a-n)}{a}$	$\frac{(a-2n)}{(a-n)}$	$\frac{(a-n)}{n}$	* $\frac{a}{n}$

As either the $\frac{a}{n}$th or the $\frac{(n-a)}{n}$th part will serve to show the desired $\frac{a}{n}$th portion of the angle, since it is immaterial whether we measure it upward or downward, we have the fact that for any fraction, as the $\frac{a}{n}$ th part, either the $\frac{n}{(n+a)}$th or the $\frac{n}{(2n-a)}$th part of the primary radius may be used to determine it.

If the $\frac{n}{(n+a)}$th part be used, measured outward from O, then the desired $\frac{a}{n}$th part will be found by measuring downward from A; if the $\frac{n}{(2n-a)}$th part, the fraction of the angle will be found measuring up from B.

IV. Further interesting relations may be discovered by setting the instrument to one arc and striking across another, the radii of the two arcs being known, the subdivision of the angle, thus determined, to be found in terms of the assumed radii.

We have the following cases, Fig. 9, $\angle$ LOD being any assumed angle to be subdivided: (1) By turning the instrument through

point Q, and striking a spiral arc across FH, which is drawn with any assumed radius, known in terms of OA. (2) By setting at E of θ'', and striking across FH of θ'. (3) Setting to R, and striking

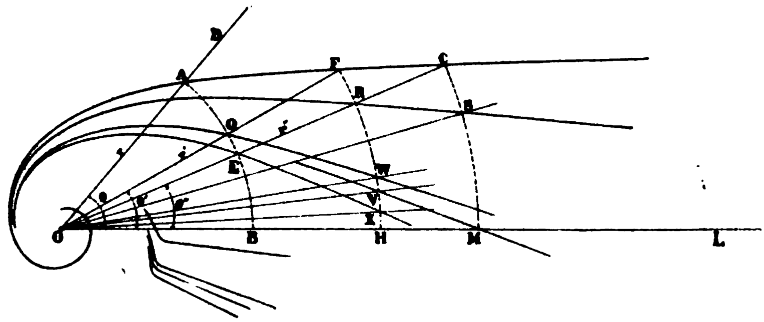

Fig. 9.

across arc CM of θ''. (4) Setting to M, and striking an arc across FH of θ'.

(1) Set at Q, and strike arc at W. Draw WO and find $\angle$ HOW in terms of θ, and r and r',

$$\frac{\theta'}{\theta}=\frac{r}{r'}=\frac{OB}{OH}.\quad \angle AOF=\angle FOW. \quad \therefore\ \theta-\theta'=\theta'-\angle HOW.$$

$$\text{Then } \theta'=\frac{\theta+\angle HOW}{2}, \text{ and from above } \frac{r}{r'}=\frac{\theta+\angle HOW}{2\theta}.$$

$$\text{And } \angle HOW=\left(\frac{2r-r'}{r'}\right)\text{th part of } \angle\theta.$$

(2) Set the instrument at E, where CO of θ'' cuts arc AB, and strike arc SX, finding X. XO determines some fraction of θ, which can be found in terms of r, r', and r''.

The $\angle$ EOA, through which the instrument turns, $=(\theta-\theta'')=$ $\angle$ FOX. Hence the fraction of θ, determined, $\angle HOX=\theta'-(\theta-\theta'')$, which $=(\theta'+\theta'')-\theta$. And the ratio of this to θ, or the part expressed as a fraction of θ, is

$$\left(\frac{\theta'+\theta''}{\theta}\right)-1, \text{ or } r\left(\frac{1}{r'}+\frac{1}{r''}\right)-1.$$

(3) Set through R and get point S. To find relation of $\angle$ MOS to θ. $\angle ROF=(\theta'-\theta)=\angle COS$. So $\angle SOM=\theta''-(\theta'-\theta'')=(2\theta''-\theta')$. Or

$$=\frac{(2\theta''-\theta')}{\theta}\text{ part of } \angle\theta, \text{ or } =r\left(\frac{2}{r''}-\frac{1}{r'}\right)\text{ part.}$$

(4) Set at M and draw OV. Get $\angle$ HOV.

$$\angle \text{HOV} = (\theta' - \theta'') = \left(\frac{1}{r'} - \frac{1}{r''}\right).$$

And ratio to θ, $\qquad = r\left(\frac{1}{r'} - \frac{1}{r''}\right).$

These relations, and others, may be derived readily from properties of the curve expressed by its equation $r = \frac{a}{\theta}$, but none of them appear to be of a sufficiently simple form to be of any further interest.

CONSTRUCTION OF REGULAR POLYGONS.

The spiral iustrument affords an interesting means for constructing the regular polygons. There are two cases: (a) The circumscribing circle given to construct a regular polygon of any required number of sides. (b) Having given the length of one side, to construct upon it a regular polygon of a required number of sides.

(a) For this case we have evidently again merely the division of an angle into a desired number of equal parts. Bearing in mind that the angle to be divided is here 360, and hence that the two sides coincide in OL, Fig. 10, any desired inscribed polygon can

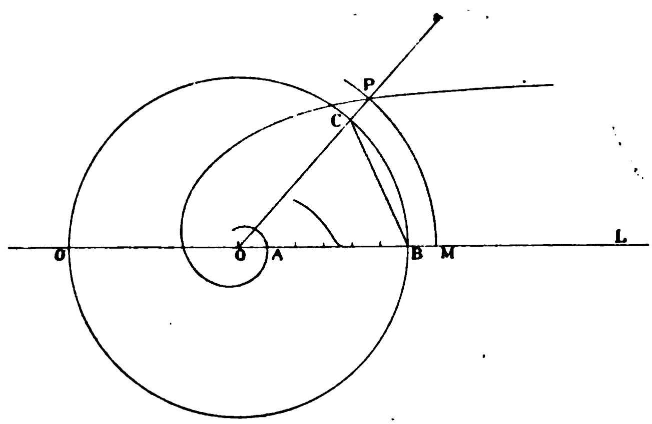

Fig. 10.

be had at once, as follows: Draw the given circumscribing circle, apply the instrument and mark A. Then lay off OA from O, as many times as the polygon is to have sides, say seven, here, obtaining point M. Then draw the arc MP, and PO determines one side of the polygon, BC, which can now be set off the remaining number of times around the circle.

Or if we have at hand an instrument of which the constant equals 360, simply set to OL and strike a spiral arc. Then with a radius as long in inches as the number of sides which the polygon is to have, strike a circular arc cutting the spiral and join the intersection to the center when the required side is determined.

If instead of a 360 degree instrument an 180 degree one is available, set the pole at O″ and lay out the angle $\left(\frac{180}{n}\right)$, when the result will be the same. Or if desired, O″ may be used in any event and $\left(\frac{180}{n}\right)$ laid out in the usual way.

(b) To construct a required polygon on a given length of side, AO, Fig. 11. We know that for any polygon of n sides,

$$\angle \text{LOP} = \frac{360}{n}.$$

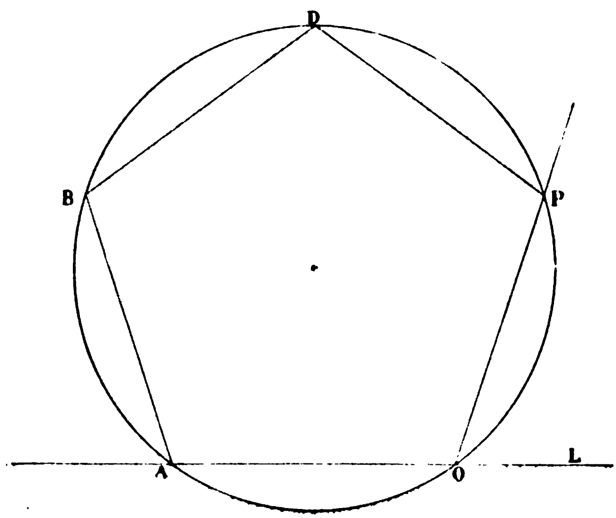

Fig. 11.

So it is a simple matter to lay out this angle $\left(\frac{360}{n}\right)$ by use of the instrument, just as in the preceding case, by either of the ways noticed there. Then, OP known, the circumscribing circle can be drawn and the polygon completed.

GRAPHIC RECTIFICATION OF ARCS.

By means of the Universal Drawing Curve the graphic rectification of circular arcs is also readily possible. We may have either

(1) a definite arc given to find a length of straight line equal in length to it, or (2) a given length of line to find an arc of assumed radius equal in length to the given line.

The constructions for these cases depend upon the fact that any arc drawn about the pole of the spiral as a center and limited by the initial line and the spiral itself is equal to every other arc so drawn. That is, in Fig. 9, arc CM = arc FH = arc AB. And this constant length of arc is known, being the constant c of the instrument multiplied by $\frac{\pi}{180}$. In the case of an instrument designed for θ to be measured in radians, instead of in degrees, it is equal the constant term of the equation.

On each of the Universal Curves will be found a fine mark along the edge of the instrument which is the initial line, shown at a, Fig. 5. This mark is accurately laid off a distance from the pole of the instrument equal to the constant length of arc of the particular size of curve. In the case of the size $r=\frac{100}{\theta}$, this length is $100\times\frac{\pi}{180}=1.745$, or very closely $1\frac{3}{4}''$.

(1) To rectify a given arc. Let AB, Fig. 12, be the given arc of which the center is O. Set the instrument as shown, marking a

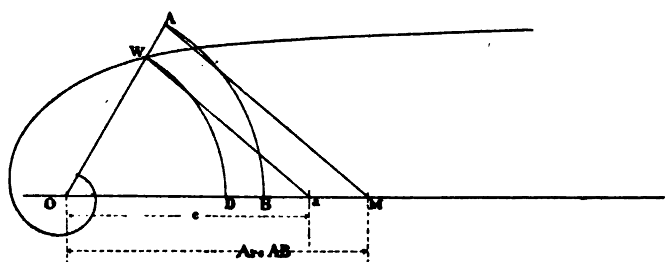

Fig. 12.

and W. Join W to a, and draw AM from A parallel to Wa, determining M. Then OM equals the length of arc AB.

For $\frac{\text{arc AB}}{\text{arc WD}}=\frac{\text{OA}}{\text{OW}}$, which by similar triangles $=\frac{\text{OM}}{\text{Oa}}$. But Oa = the arc WD by the property of the instrument.

$$\therefore \text{ OM} = \text{arc AB}.$$

(2) Given a length of line to find an arc of given radius equal in length to the line.

In Fig. 13, let BPA, etc., be an indefinite arc of the given radius. Lay off OM equal the given length of the arc to be found, and mark the point a. Now join any point of the arc drawn, as P to M.

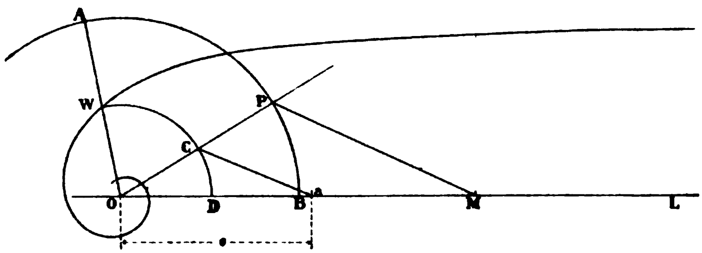

Fig. 13.

Draw aC from a, parallel to MP, thus determining C. With radius OC determine W, and produce OW to A. Then AB is the required arc.

For $\frac{AB}{WD}=\frac{OP}{OC}=\frac{OM}{Oa}$. But arc WD=Oa, by the property of the instrument.

$\therefore$ Arc AB=OM.

MECHANICAL PROPERTIES OF THE SPIRAL INSTRUMENT.

Although the mathematical properties noticed are interesting, and some of them make possible a number of drawing board operations by use of the instrument described, their bearing from the standpoint of actual utility is not to be compared to that of the curve's physical characteristics. The great value of the Hyperbolic Spiral lies in the fact that it possesses just the proper rate of variation and range of curvature to make it the ideal outline for the drawing instruments commonly known as "irregular curves."

Irregular Curves.

Irregular curves, so-called, are used by draftsmen for drawing smooth curves through ranges of points on a drawing determined by plotting or otherwise, where these points locate some curve of definite character not circular. These instruments are made by cutting out a variety of curves, usually of no definite mathematical character, from uniform thin sheets of pearwood, hard rubber, transparent celluloid or metal, as may be desired. The outline of one of these instruments usually consists of many kinds of curves of different degrees of curvatures, chosen either entirely at a venture, or perhaps from some experience with curves of approximately the same form. Often the ornamental appearance has more influ-

ence than any other consideration in determining the outline used.

Consequently, as a rule, any one irregular curve is suitable for only a limited use. So that a draftsman must have a number of these with him, and often must try his whole assortment several times before being able to fit a comparatively simple curve. The exasperating nature of this task is only appreciated by those who have had some bit of curve on an important drawing to put in with great accuracy.

The Universal Drawing Curve.

The very great desirability of having one instrument which can be used easily for all plottings within reasonable limits of size suggested the design of the Universal Drawing Curve, which is the same instrument, some of the mathematical possibilities of which have been noticed. Fig. 14 shows a photograph of six sizes of the instrument as now made, reduced about one-fourth size. Any desired size may be made, possessing the same characteristics, by simply varying the constant in the plotting of the spiral.

Continued use for a long time under a very great variety of conditions shows the instrument to be truly a Universal Curve. It possesses such a wide range of curvature, varying, as it does, from nearly a straight line down to a very short and almost circular curve, including all degrees of curvature ever needed, that with one of these instruments it is a simple matter to fit any kind of plotting within a reasonable limits as to size, to allow for which the instrument is made in this series of sizes. To do this it is only necessary to turn the instrument on the drawing till the curve of the spiral coincides with three or more points of the plotting, draw a line through these points, and then turn the instrument a little farther until a small portion of the line already drawn and several more points are fitted. In this way the line is quickly prolonged without the usual exasperating work of picking up and turning over a whole assortment of curves in order to fit a small bit of curvature at a time.

Either of the medium sizes, $c=100$, $c=120$ or $c=135$ is admirably adapted to average work. The size $c=60$ is for exceptionally small work, and $c=180$ and $c=225$ for very large work. Use shows that any one of these instruments will replace a whole assortment of the old curves and enable a draftsman to do his work quicker and better.

The Sacrum of Morosaurus.

Contributions from the Paleontological Laboratory, No. 33.

BY S. W. WILLISTON.

A recent paper by Prof. H. F. Osborn on "Some Additional Characters of the great Herbivorous Dinosaur Camerasaurus"* shows with much reason that the number of sacral vertebrae in the Camerasauridae is a character not valid in the separation of genera. This opinion I have long had from a knowledge of the type specimens upon which Marsh's genera were based. It is very clear that there are three typical sacral vertebrae in all the genera of this family, as well as in the Morosauridae, if it be a distinct family, all of which present very distinct points of similarity. It is probable, as evidenced by the separated sacral vertebrae in *Morosaurus lentus*,† that the condition of ossification varies with age, the middle three uniting earliest, the first next and the fifth last. The slight union of the fifth might, indeed, be absent in the adult without affording generic or even specific characters.

In the University of Kansas Museum there is a portion of a skeleton of a species of *Morosaurus*, doubtless *M. grandis*, obtained by the expedition of 1895, from Converse Co., Wyoming. Among the different bones there is a sacrum evidently in much better condition than any hitherto made known in the Cetiosauria, though not complete. It has four vertebrae united firmly, agreeing well with the sacrum figured by Marsh (The Dinosaurs of North Amer-

*Bull. Amer. Mus. Nat. Hist., x, pp. 219-233.

†The various species of this genus described so far are as follows:
Morosaurus Marsh, Amer. Journ. Sci., xv, p. 242. 1878; xvi. p. 412, 1878.
grandis Marsh, op. cit. xv, p. 514, 1877 (*Apatosaurus*); xvi, p. 416, pls. v, vi, vii. Feb, 1879, pl. xvii.—Wyoming.
impar Marsh, op. cit. xv, p. 242 1878 (this species is clearly identical with the preceding).—Wyoming.
robustus Marsh. op. cit. xvi. p. 414. 1878.—Wyoming.
lentus Marsh, op. cit. xxxvii, p. 332, 1889.—Wyoming.
agilis Marsh. op. cit. xxxvii, p. 333, 1889.—Colorado.

ica, 16th Ann. Rep. U. S. G. S., pl. xxi, f. 8) and doubtless is conspecific with it. A front view of this specimen is shown in the accompanying figure (Fig. 1), together with a photographic reproduc-

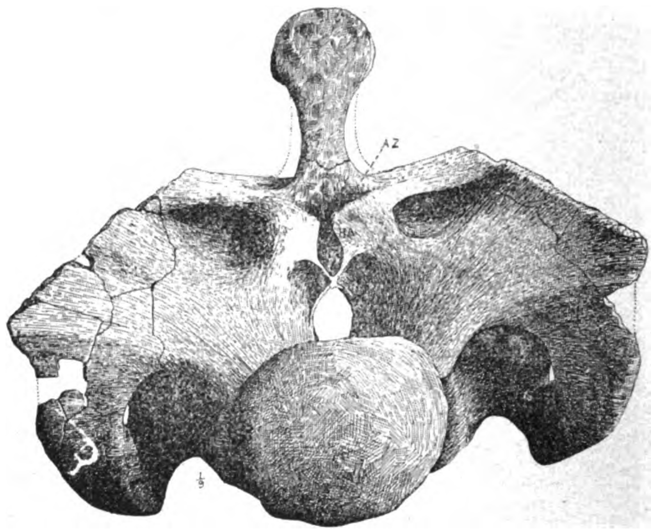

Fig. 1. Sacrum of *Morosaurus grandis*, from in front, *AZ*, anterior zygapophyses; *HA*. hypantrum.

tion of a side view (Fig. 2). It will be observed that the first vertebra takes but little part in the iliac articulation and that the transverse processes of this vertebra arise much higher up than those of the following. The broad plate in front seems to represent in its upper bars, which are thickened, the transverse processes of the posterior dorsals. Marsh has figured in plate xxxii of the work cited what he believes to be a posterior dorsal of this species. It is very evident, however, that there are a number of vertebrae intervening between that and the one immediately preceding the sacrum. In all probability, as Osborn has suggested in *Camerasaurus*, there will be found a larger number intervening here than has been hitherto supposed, the posterior ones partaking more of the characters of the first sacral, even as it has been shown that the pygal caudals present the prominent characters of the last sacral.

On plate xxxiii of the work cited, March has figured the disconnected sacral vertebrae of *M. lentus*, I am confident that there is something wrong in their interpretation. It is impossible that the vertebra there considered the first can be the same as the first of

the accompanying figures. Its flat anterior surface and the shape of the transverse processes are very different.

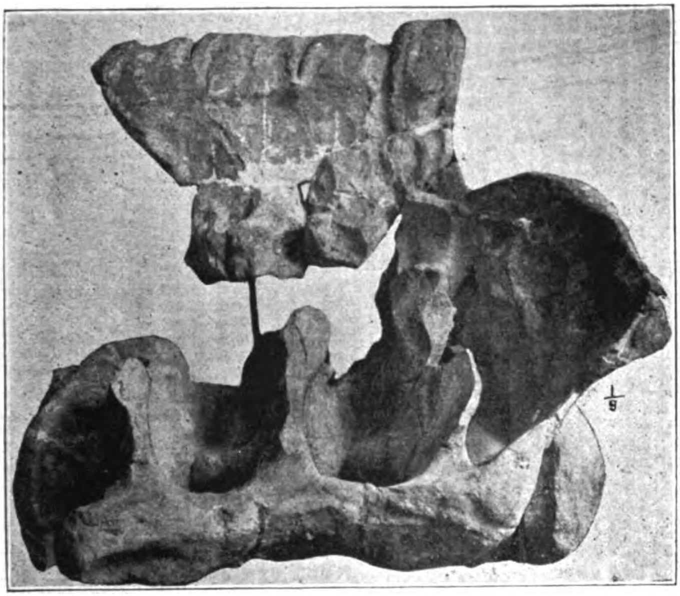

Fig. 2. Sacrum of *Morosaurus grandis*, from the side.

On either side of the broad plate, at the extreme ends, there is a thickening, tipped with cartilage. The same protuberance is found on the much smaller plate of the second vertebra. It is apparently wanting in the succeeding vertebrae, whose plates are much reduced. The middle three spines of this specimen, as in the other genera, are firmly coossified. The first vertebra is closely united as far as the bottom of the spine, but the spine itself is free.

The generic identity of the present specimen is assured by the finding of the complete ischia in immediate conjunction with the sacrum. Aside from a number of dorsal, cervical and caudal vertebrae, all clearly belonging with the same individual, there were no other bones in the "quarry" whence the specimen came.

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Some Notes on the Genus Saurodon and Allied Species.

Contribution from the Paleontological Laboratory No. 34.

BY ALBAN STEWART.

With Plates XIV, XV, XVI.

In the year 1824, Dr. Harlan* described the genus *Saurocephalus* from a fragment of a superior maxilla collected by Lewis and Clark in their expedition up the Missouri river in 1804. Six years later, Dr. Hays† described the jaws and a portion of the skull of a second form *Saurodon leanus*, from the Marl of New Jersey. He also examined the specimen described by Dr. Harlan and found "that the teeth, instead of being 'in a longitudinal groove' 'in close contact throughout' 'there being no distinct alveoli,' *are in fact placed in distinct alveoli.*"‡ Dr. Hays then reached the conclusion that the two genera were synonymous, and that since Dr. Harlan's genus was founded upon erroneous characters, the name Saurodon should take precedence over it. In the year 1856, Dr. Leidy redescribed both of the above genera in a paper read before the American Philosophical Society,|| and as there had been nothing new added to the knowledge of them in America up to that time, concluded that the name Saurodon should be abandoned, and that the type of this species should be known as *Saurocephalus leanus* Hays. Nothing further was done regarding either of the generic

*Jour. Acad. Nat. Sc., Phila., 1830, vol. iii. p. 331.
†Trans. Am. Phil. Soc., vol. iii, p. 471.
‡L. c. p. 476.
||Trans. Am. Phil. Soc., vol. xi, p. 91.

terms until 1875, when Prof. Cope* added the allied genus *Daptinus* from the cretaceous of Kansas, but which he later recognized as a synonym of *Saurodon* Hays.

The exact date of Prof. Cope's retraction I have been unable to exactly determine, but it was probably not until after 1878, as at this time Mr. E. T. Newton† described a fish from the lower chalk of Dover and provisionally placed it in *Daptinus*. A little later, in the same year, Mr. Newton published a paper entitled‡ "Remarks on *Saurocephalus*, and on the Species which have been referred to that Genus." In this paper Mr. Newton carefully goes over the ground and finally coucludes, as Dr. Leidy had already done, that the name Saurodon should be no longer used.

After carefully examining the material at my command I am led to the conclusion that there are two distinct genera, which should be known as *Saurocephalus* and *Saurodon;* the evidence for which the following descriptions, I think will make apparent.

During the past summer the museum was presented with a fine specimen of *Saurodon* by Mr. H. M. McDowell of this place, and was also loaned another specimen of this genus by Mr. W. O. Bourne of Scott City. The second specimen is not so complete as the first yet it shows many points of interest that are not visible in the first specimen mentioned. The two forms are new to science and will be called *Saurodon xiphirostris* and *Saurodon ferox* respectively.

Saurodon xiphirostris sp. nov.

This specimen consists of a skull crushed obliquely, the centra of several vertebrae, and also a portion of a shoulder girdle in a very bad state of preservation. The specimen is of nearly the same size as that of the type of *Saurodon* (*Daptinus*) *broadheadii*‖ described by myself.

The maxilla is short and deep, the depth not being as great as in *S. broadheadii*. The alveolar border is nearly straight and has alveoli for about thirty-one teeth, which are about the size of those described in the above species. The posterior extremity can not be examined as there is a suborbital bone covering this portion on each side of the skull, but it is probably very similar to that of the figures of *Saurodon ferox* described below. The superior border is sharp, and gives attachment to some bone, probably a sub-orbital or jugal. The palatine condyle seems to be very

*Cret. Vert. West., p. 213.
†Quart. Jour. Geol. Soc., 1878, No. 135, p. 439.
‡Quart. Jour. Geol. Soc., 1878, No. 136, p. 786.
‖Kans. Univ. Quart. vol. vii, p. 21-29.

similar to that of *S. broadheadii* already described. The bone, just above the alveolar border, presents a somewhat shagreened appearance. Farther than this, there seems to be no characteristic markings upon the external surface. As the maxilla is firmly attached to the skull, the internal surface can not be examined.

The premaxilla is plate-like, and nearly twice as deep as broad. The superior border is irregular and presents no condyle at this point. The external markings are very similar to those found in *Saurocephalus dentatus*, and the bone is directed more obliquely backward than in that form. The anterior border is directed sharply inward, giving the external surface of the bone a very convex appearance. This border is very rugose, probably for ligamentous union with its fellow of the opposite side. The alveolar border is very convex and has alveoli for twelve teeth, which are of about the same size as those on the maxillae.

MEASUREMENTS OF MAXILLA AND PREMAXILLA.

		mm.
Maxillary:	length of alveolar border	82*
"	depth at palatine condyle	33
"	number of teeth in 1. c. m.	4
Premaxillary:	greatest depth	50
"	greatest length	30
"	length of anterior border	30

In the mandible is found one of the chief characters that separate this genus from *Saurocephalus*; instead of the upper and lower jaws terminating at about the same vertical plane as in all other members of the *Saurodontidae*, the mandible projects fully an inch beyond this point. The dentary is long and slender throughout, in *Saurocephalus* it is short and deep. This difference is well illustrated by comparing the types of *Saurocephalus dentatus* and *Saurodon ferox*; the maxilla of the first is considerably longer than that of the second, but with the dentaries, the reverse is the case. Only a small portion of the dentary can be seen, as most of the external and superior portions are hidden by the overlying maxillae. The bones are irregular and shallow at the symphysis and seem to have given strong attachment for the predentary. The lower border is thin and sharp. Only twenty-seven millimeters of the alveolar border can be seen in the specimen upon which the teeth are small and twelve in number. At the base of each tooth is found the deep notch, for the nutrient vessels, so characteristic of this genus. As the articular portion does not seem to differ materially from that of *S. ferox*, its description may be deferred.

*Estimated.

Contrary to anticipations there is but one predentary, as is proven by the discovery of all of the parts in place. It is long and slender, triangular in outline, with a broad elliptical articular surface at the posterior extremity. When this element was first made known* in this genus, I was under the impression that it was paired, which is not the case. This slender projection was no doubt used as a weapon of offense, analogous to the rostrum of *Protosphyraena*. In connection with the description of *Saurodon broadheadii.*† I figured a predentary of an enterely different form from the above, which was found on the same slab with the maxilla described; the form is the same as that found in *Saurocephalus*. Whether the two bones belonged to the same individual or not, only future discoveries can determine. After carefully comparing the type of *S. dentatus* with that of the species under consideration and *S. ferox*, I am convinced that there is but one predentary in the mandible of this form, as one would expect from the great similarity of the two genera.

MEASUREMENTS.

	mm.
Mandible length from cotyloid cavity	155
" depth at symphysis	23
" number of teeth in 1., c. m.	4.5
Predentary; length	73‡
" depth of symphyseal surface	23
" width of symphyseal surface	12

The ethmoid is broad and flat posteriorly, becoming thickened and pointed at the anterior extremity. The lower surface can not be seen, but it probably is not materially different from that of *Ichthyodectes*. In Prof. E. T. Newton's description of *S. intermedius*|| he says, concerning this part; "Anterior to the frontals, upon the upper surface of the skull, there are two bones (fig. 2) separated by a median longitudinal suture; towards the front of these an osseous band passes across at right angles, obliterating the suture." In our skull I am unable to detect any indication of a suture at this point such as is shown in the figure referred to above. I have also examined all of the specimens of *Xiphactinus* and *Ichthyodectes* in the museum, and find no trace of such a suture in any of them. It seems probable to me that the skull, described by Prof. Newton, was a younger individual than are any of ours.

*Kans. Univ. Quart., vol. vii, p. 24.
†L. c. pl. ii.
‡Estimated.
||Quart. Journ. Geol. Soc., vol. xxxiv, No. 135, p. 441, pl. xix.

The frontals are broad, flat bones extending from the ethmoid, with which they are united by a squamose suture, back to the parietals. Laterally, they form the superior borders of the orbits. In the median line they are separated by a suture. The bones are probably in contact with the supraoccipital, but, owing to the crushed condition of this region, this point can not be definitely determined.

The parietals are small elements, in contact with the pterotics and the epiotics (?) posteriorly. The epiotics are probably coössified with the parietals; at the point where the suture should be, there is a crack, but the edges at no point show signs of sutural union with the opposite portion. If this be the case, the parietals nearly or quite meet in the median line. If the bones are not coössified, the parietals are separated by a long slender anterior process of the epiotic. There seems to be a faint suture between the parietals and the sphenotic. The epiotic process of the parietal* does not seem to be produced as far posteriorly in this species as in *S. intermedius*, described and figured by Newton.† They are heavy projections of bone and form the inner lateral processes of the skull as in other members of the *Saurodontidæ*. Mr. Newton‡ seems somewhat in doubt about the bones in this region, and is unwilling to accept the bone called parietal by Cope, stating for his reason that the lines indicative of the direction of growth were from the extreme posterior angle of the skull instead of from the anterior portion as we would expect if this bone were the parietal. In a skull of *Ichthyodectes*, before me, there is a distinct suture between the parietal and epiotic, and in this specimen the lines, indicating growth, radiate from the posterior portion of the bone as in *Saurodon intermedius* and the specimen under consideration. I also find the same condition in skulls of *Xiphactinus* which I have examined.

The pterotics are large bones. The line of separation between them and the parietal can not be traced throughout, but this is probably due to the crushed condition of this bone. The radiating lines pass upward and backward instead of upward and forward as in *S. intermedius* Newton. It seems to be very dense in structure.

The supraoccipital is very much crushed and broken away, but enough remains to show that the bone was raised into quite a prominent crest It extends backward beyond the points of the

*I think it should be called thus until the presence or absence of the suture between this bone and the parietal is definitely determined.
†l c., p. 444, pl. xxxiv.
‡l c., p. 444-445.

epiotics, a condition different from that described by Newton.* It may join the frontals in front, but a small portion of the parietals may intervene.

The orbit is somewhat smaller than in *Ichthyodectes* and is surrounded by a thin ring of sclerotic bones similar to that found in the genus just mentioned. Just in front of the orbit there is a small triangular bone attached to the frontal above, which I take to be a preorbital. The same bone is figured by Newton† but not named or described by the author. Just in front of this there is a long slender bar of bone, which seems to articulate somewhere in the palatine region. On one side the anterior end is crushed down to near the posterior condyle of the maxillary, but on the other side it fits in just back of the superior condyle of the palatine, and as a palatine of another specimen shows a sutural surface at this point, I think it not unlikely that this is the correct position of the bone. This may be the bone that Newton† figured as a "nasal bone," (?) although it is of an entirely different shape from that shown in the cut of his specimen. The bone Newton calls "jugal"(?) I am inclined to think is one of the suborbital bones, as found in *Xiphactinus*, especially as there seems to be a suture indicated between this and a bone just above which articulates with the jugal, (probably a suborbital) above.

Owing to the crushed condition, the prefrontals are almost entirely covered by the ethmoid and frontal. The description of the palatine will be given with the next species. Parts of the opercular and preopercular bones are present. The first is a broad flat plate of bone which articulates with the hyomandibular in a manner similar to that found in *Xiphactinus* and *Ichthyodectes*. The anterior border of the preoperculum is deeply concave, the anterior inferior extremity reaching forward to the angle of the mandible. The hyomandibular of this species is not visible.

The vertebræ are deeply concave, with deep groves closely situated above for the neural arches. The ribs articulate with small ossicles set into deep pits on the side of the centrum. Just above these ossicles there is a deep pit on each side.

A part of the shoulder girdle, including a fragmentary fin, is present. The fin seems rather small.

The skull as a whole is especially remarkable for the extreme length of the mandible, and also the long predentary in front. This portion probably had a dermal covering similar to that cover-

*l c., p. 444.
†l c., pl. xxxiv.

ing the sword-fishes' sword and was no doubt used as a weapon of offense. In an animal with such a weapon as this, we might expect to find powerful fins, but this is not the case with this species. In other respects the skull does not materially differ, excepting in details, from the skull of other members of the *Saurodontidæ*.

Saurodon ferox sp. nov.

This species is represented by the jaws, including the predentary, and other disarticulated bones and vertebræ.

The maxillary is larger than the one just described. The posterior condyles above are somewhat elliptical in outline and but slightly convex from before backwards. Just anterior to this there is a large protuberance which may support a condyle above, and on the external side of this there is a small facet which probably gives articulation with the ethmoid. The surface for the premaxillary is very irregular and is directed inward, becoming thinner toward the anterior border which is sharp. The superior border is strongly concave and sharp, and presents a sutural surface on the external side probably for a supernumerary bone or jugal. The alveolar border is convex and has alveoli for forty or forty-one teeth which are non-striate and knife-like. Each tooth has the characteristic notch at the base. The posterior extremity of the bone is very shallow and turned slightly upward. Aside from the shagreened surface of the bone above the alveolar border the external surface has no characteristic markings.

The premaxmillary is very similar to that of the species described above, except that there are ten instead of twelve teeth. On the internal side the bone is bevelled off toward the posterior border in order to fit the surface for its reception on the maxillary.

MEASUREMENTS OF MAXILLARY AND PREMAXILLARY.

Maxillary:	length of alveolar border	115
"	depth at condyle for palatine	45
"	greatest length of bone	128
"	number of teeth in 1 c. m	3.5
Premaxillary:	depth	60
"	length	32

The dentary is elongate and slender. The alveolar border is slightly incurved at the sympyhsis and supports forty-six teeth, similar in form but about twice as large as those upon the maxilla. Just back of the last tooth there is a slight coronoid process, somewhat similar to that found in *Xiphactinus*. The symphysis is very similar to that found in the last species described, and has

a long slender pit just back of it on the internal side. It is more elliptical than the corresponding pit in *Saurocephalus*. The lower border of the bone is sharp.

In a paper recently published by Prof. Hay *U. S. N. M. the author corrects some errors in the former descriptions of the jaws of *Xiphactinus*. I have for sometime been of the opinion that the bones of this region had not been correctly interpreted, and as the arrangement of the articular portion of this genus, seems to agree with the figure and description of this part of *Xiphactinus* as given by the author, I think it well to adopt his nomenclature.

The dermarticular supports only a small portion of the cotylus, in the lower portion of the cavity. It sends a long sword-shaped process forward internally, but does not encroach much upon the dentary externally. Posteriorly, it sends a lamina of bone backward, which I think would be well named the cotyloid process as in most of the genera of this family; it is the only portion of this bone which articulates directly with the quadrate. Just beneath the cotyloid process, there is a prominent angle.

The autarticular is a small element not extending forward beyond the sixth posterior tooth, and is fitted into a groove in the dermarticular. It supports nearly the whole of the cotylus, which is somewhat elliptical and concave from above downward. The predentary is not so elongated but is slightly deeper than in the species described above.

MEASUREMENTS OF MANDIBLE.

		mm.
Mandible;	length of alveolar border	149
"	length from cotyloid cavity	174
"	depth at symphysis	27
"	depth at coronoid	46
"	number of teeth in 1. c. m	3
Predentary;	length	55
"	length of symphyseal surface	24

The quadrate is a broad fan-shaped expansion. The condyle is elliptical in outline and convex. Extending upward from the condyle on the external side there is a ridge which ends above in a deep notch which accommodates a portion of the sympletic. This groove continues downward on the internal side for more than one-half the depth of the bone. The posterior border has a very slight groove and extends upward the whole extent of the sympletic. The superior border probably articulates with the pterygoid and metapterygoid as in *Xiphactinus*. The anterior border is sharp. Both the external and internal surfaces are covered with

*Zoological Bull., vol. II, No. 1, pp. 86-88.

minute striæ radiating upward from the condyle. The sympletic is a long slender element. The upper end presents an articular surface, similar to that found on the superior border of the quadrate.

The whole of the palatine is preserved. It is an irregular-shaped bone, presenting a ragged sutural surface above and below for the pterygoid and mesopterygoid. The bone is especially remarkable for the great depth of the maleolar portion, being nearly half as deep as the corresponding part in *Xiphactinus*. The superior articular surface is small and oval in outline, while the lower is larger and more elliptical. The depth of the maleolar portion is 20 mm.

The hyomandibular is very similar to that found in *Ichthyodectes*. The superior condyle is elongated and depressed in the central portion. In the skull of *S. intermedius*, figured by Newton*, this condyle is shown to be regularly rounded from before backward. As all the figures and descriptions of the Saurodont hyomandibular show the depression described above, I am inclined to think that this portion may have been distorted in the specimen figured by Mr. Newton. Extending downward from the anterior and posterior angles, there are two slight ridges which converge toward the center and form a much larger one which extends downward to nearly the lower extremity of the bone. It forms the outer border of the groove for the preoperculum. There is also another ridge on the internal side, just in front of the condyle for the operculum, but it is not so prominent as the one just mentioned. The condyle for the operculum is elongated as in *Ichthyodectes*.

The lower extremity of the bone presents an articular surface similar in size and character to that found at the upper end of the sympletic. I think it is very likely that these two bones articulate at this point.

A small portion of a scapula is preserved. It shows only two distinct articular condyles, one large and one small, instead of three as in *Xiphactinus*. Portions of several spines are preserved, of which one complete and a portion of another are shown in the figure and need no further consideration.

The first anterior vertebra has the posterior end deeply concave, but the anterior end is not so deep, and has a slight protuberance projecting forward above. On the superior surface there are two deep, rounded pits for the neurapophyses; aside from this there are no other grooves displayed upon the centra.

A small toothed element was found on the internal side of one of the maxillæ at about the point where the pterygoid should lie, but

*L. c. pl. xxxiv.

it is too large to be a portion of this bone. There are nine teeth upon it, which are about the size of those found upon the anterior portion of the maxilla.

Below are given the known American species of this and the related genus *Saurocephalus*:

Saurodon leanus Hays, Marl, New Jersey.
" **phlebotomus** Cope, Niobrara Cretaceous, Western Kansas.
" **broadheadi** Stewart, " " " "
" **xiphirostris** Stewart, " " " "
" **ferox** Stewart, " " " "
Saurocephalus lanciformis Harlan, Cretaceous, Missouri River.
" **arapahovius** Cope, Niobrara Cretaceous, Western Kansas.
" **dentatus** Stewart, " " " "

Lawrence, Kansas, Oct. 6, 1898.

Notes on Campophyllum torquium Owen, and a new Variety of Monopteria gibbosa Meek and Worthen.

Contributions from the Paleontological Laboratory, No. 35.

BY J. W. BEEDE.

Campophyllum torquium Owen.

"Corallum simple, attaining a rather large size, elongate conical, and often variously geniculated or bent when two or three inches in length, but becoming nearly straight, subcylindrical, and considerably elongated in the larger half of adult individuals. Epitheca thin with small encircling wrinkles and strong undulations of growth, showing no traces of the septal costæ when unabraided, but, where even slightly worn, exposing the regularly disposed septa and thin intervening dissepiments distinctly. Calice circular or slightly oval, comparatively shallow, with thin margins, from which its sides slope rather steeply inward for some distance and then descend very abruptly into a deeper, narrow, central depression; provided at the outer side of the general curve of the corallum with a moderately distinct septal fossula, formed by the shortening of one of the primary septa, and the bending down of the tabulæ at that point. Principal septa from 30 to 48, extending from about one-half to two-thirds the distance from the exterior toward the center, stout and usually straight inside of the outer vessicular zone, but becoming distinctly more attenuated (as seen in transverse sections) and somewhat curved or a little flexuous in crossing the vesicular area, where they alternate with an equal number of very short, thin ones; tabulæ very wide or occupying about two-thirds the entire breadth as seen in longitudinal sections, and passing nearly hortizontally across with a more or less upward arching; dissepiments thin and forming numerous obliquely ascending, small vesicles, in transverse sections seen to pass across

between the costae with an outward curve. Entire length unknown, but individuals incomplete in both extremities, 5 inches in length and 1.60 inches in breadth have been met with. Individuals of this size show at the thickest part 9 costæ in a space of .50 inches."*

In addition to the above characters may be mentioned: In young specimens the cardinal septum and all the other septa on that side of the corallum are very greatly developed laterally after passing inward from the vesicular zone; the inner wall of the vesicular zone is also thickened on that side of the corallum. As a result of this great thickening of the septa the interspaces are small, producing a peculiar appearance in cross sections as shown in figure 1. The septal development becomes less and less marked as the specimen advances in age until in old specimens it is hardly noticeable, save in the cardinal septum, though a close comparison generally shows them a little larger on the outside of the general curve. These peculiarities may be seen by sectioning the large and small ends of any adult specimen. The thickness of the dissepimental zone and also the number of tabular dissepiments are variable, as will be seen by comparing figures 2 and 3 with those of Meek.† Some specimens in the collections of the University are about 9 inches long and apparently incomplete at both ends. Such specimens are generally a litlle crooked throughout their entire length. The young specimens are either rather slender or quite turbinate; the latter form is shown in figure 4, which is twice natural size. Through the courtesy of Mr. Gharles Schuchert of the U. S. National Museum I have had the pleasure of examining some specimens from Rock Bluff, Nebraska, belonging to the collection studied by Meek, and upon which he based his description. These show the above mentioned pecularities quite distinctly. Specimens from Jefferson, Douglas, and Chatauqua counties, Kansas (all from about the horizon of the Lecompton Limestone of Bennett), show these characters equally well, as does also a specimen collected in a quarry in "Northrop's Woods" about three miles west of Kansas City, Kansas. The horizon of the Rock Bluff locality is probably above that of the Lecompton limestone. It is evident that the species has a considerable geological as well as geographical range in Kansas.

Among the many interesting fossils which the Turner Oölite has produced are two right valves of a *Monopteria* apparently quite distinct from any species I have yet seen described.

*Meek. Fin. Rep. U. S. G. S. Neb. 1872, p. 145, pl. 1. Fig. 1.
†Loc. cit.

Monopteria gibbosa alata. var. nov.

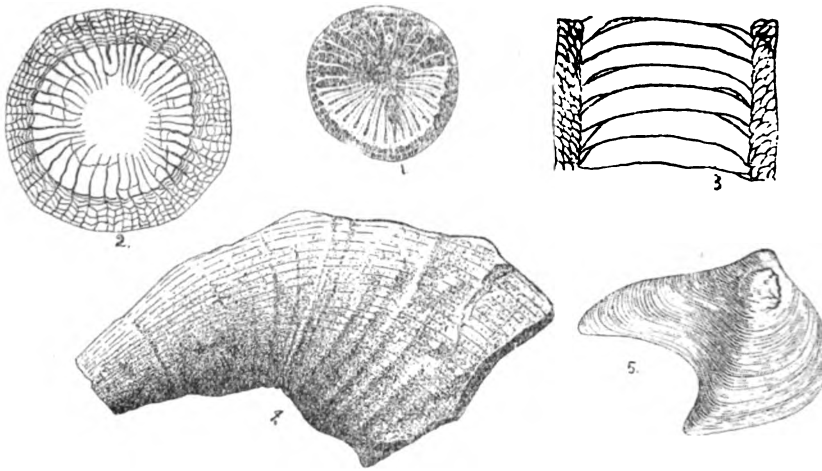
Shell small, not very gibbous; beak extending a trifle beyond the hinge-line. Prominent on account of lunule, but not much elevated, and placed well back for this genus; umbonal ridge less prominent than in any other species of this genus and less curved. Posterior ear very greatly developed, about equalling the entire body of the shell in area. Antero-dorsal margin sinuate on account of the turning down of the margin to form the lunule: anterior margin circular nearly to the postero-ventral extremity of the shell, which is acute; the posterior margin consists of a broad shallow sinus, extending from the postero-ventral end to the point of the ear which is apparently rounded and obtuse. The ear is not separated from the shell by a distinct depression, but slopes gradually from the umbonal swell, save to the extreme lower edge where the depression is more abrupt. Very fine, concentric lines of growth are visible, all of which pass around the shell with a double curve to the ear, where they again curve backward, and then forward to the hinge-line. Length 20 mm., depth 18 mm., convexity of single valve a trifle less than 4 mm.

This shell differs from *M. gibbosa* M. and W. to which it is most closely related in some respects, in being much less gibbous, ear much larger and more obtuse, antero-dorsal outline more sinuous, umbonal ridge much straighter and less prominent, beak placed farther back and depression separating the ear from the umbonal ridge more shallow. These characters, if permanent, are amply sufficient upon which to base a species, but I refrain from assigning specific importance to them until more material has been studied. As a means of distinquishing the form I suggest the above varietal name.

It differs from *Monotis sp?* Keyes* in being more round on the anterior margin and more sinuate on the antero-dorsal margin; ear about twice as large, depression separating the ear from the umbonal ridge less distinct; beak placed farther back; postero-ventral extremity more acute, and the umbonal ridge a little less prominent, though of about the same degree of curvature.

In figure 5, herewith given (twice natural size), the artist has represented the hinge-line as slightly curved, which gives the beak undue prominence. The margin is embedded in the matrix and the shell is too thin and frail to admit of its removal. The artist followed the outline as it appears in the matrix.

*Geol. Surv.. Mo. vol. v. pl. xlvi. Fig. 10.



Campophyllum torquium and Monopteria gibbosa alata.

A Preliminary Description of Seven New Species of Fish from the Cretaceous of Kansas.

Contributions from the Paleontological Laboratory, No. 36.

BY ALBAN STEWART.

With Plate XVII.

The following are descriptions of some specimens of Cretaceous fishes contained in the museum of the University of Kansas, which I think are new to science.

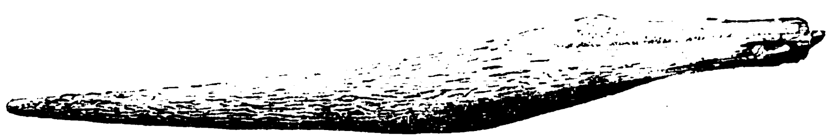


Fig. 1. Rostrum of P. recurvirostris.

PROTOSPHYRÆNA Leidy.

Protosphyræna recurvirostris sp. nov.

The material upon which this species is based consists of a complete rostrum, No. 373, with the adjoining portions of the vomer and parasphenoid, and differs from *P. nitida* in the following important characters. The superior distal surface is regularly rounded and not flat as in the species mentioned, and the cross-section at this point is round instead of semi-circular or oval. The specimen corresponds in some respects with *P. penetrans*, and to this species it is more closely allied than to any of the other forms which have been described. I was inclined to call it *P. penetrans* until I found a specimen of this species, when I discovered that it differed from it in a number of points which were sufficiently great to be called specific. The rostrum is more slender as a whole and is contracted to a more acute point than in

P. penetrans. The markings are more sharply defined and the ridges inosculate with each other but rarely. In *P. penetrans* the markings are more or less reticulate. In the anterior portion of the species under consideration the ridges are closely placed to each other, while posteriorly they become scattered and are not so well marked as in the anterior portion of the bone. The direction also becomes more varied in this region. On the posterior half of the inferior surface the ridges become less numerous and are larger than those on the upper surface and upon the sides. In *P. penetrans* there is no difference of marking on the superior and inferior surfaces. A part of one of the large teeth at the base of the rostrum is preserved. It presents a smooth enameled surface and probably had anterior and posterior cutting edges.

The space intervening between the two teeth seems to be hardly so great as in *P. penetrans*. A point that I have noticed is, that all the figures and specimens show only one tooth on this portion, the alveola for the other seeming to be filled with bone or matrix. This would lead to the belief that these teeth were alternately functional. The rostrum as a whole is recurved and from this character the name of the species, recurvirostris, is derived.

Locality; Niobrara Cretaceous, Gove county, Kansas.

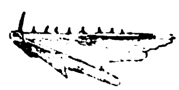


Fig. 2. Right dentary of E. parvus.

ENCHODUS Agassiz.

Enchodus parvus sp. nov.

Represented by the right dentary of a single individual, No. 321. This species differs from *E. shumardii* Leidy in having nine or ten teeth upon the internal side of the dentary, and in having the most anterior of these smaller than in this species. The specimen is small as the name indicates.

Locality; Niobrara Cretaceous, Gove county, Kansas; collector, E. P. West.

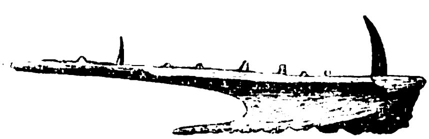


Fig. 3. Right dentary of E. amicrodus.

Enchodus amicrodus sp. nov.

This species is based upon the right dentary bone of a single individual, No. 324. The dentary is shallow and supports ten or eleven large teeth upon the alveolar border, the anterior of which is slightly recurved. The external border bears no fringe of minute teeth, and the symphysis is not so rugose as in other species of this genus.

PACHYRHIZODUS Agassiz.

Pachyrhizodus leptognathus sp. nov.

This species is based upon the left mandible, quadrate, symplectic (?) preoperculum, and two broad flat plates which probably represent two more of the opercular bones. The museum number of the specimen is 75. With the exception of the symphyseal portion of the mandible only the internal sides of the bones can be seen. The dentary is long and very slender and bears eighteen small conical teeth upon its superior border each of which is set upon a small bony tubercle similar to that found in *P. caninus*. The symphysis is more or less tubercular and is similar in many respects to *P. latimentum*. There are no teeth arranged in a triangle on this portion as in *P. caninus*. The cotylus is concave from before backwards and strongly convex laterally.

The quadrate is triangular in outline and assumes a somewhat twisted appearance towards the condyle. The condyle is somewhat bifurcated in order to fit the cotylus. Just back of the quadrate and closely applied to it there is a small element which probably is the sympletic.

The preoperculum consists of a vertical and a horizontal portion which meet each other at almost right angles. The vertical portion is broad below, becoming much more narrow above. Just back of the anterior border of this part there is a well defined ridge extending downward to the horizontal portion. The two flat bones mentioned above possess no characters other than are shown in the figure.

MEASUREMENTS.

		mm.
Mandible:	length of alveolar border	110.5
"	length of cotyloid cavity	123
"	number of teeth in one inch	5
Quadrate:	depth	32
"	length of superior border	34.5

Pachyrhizodus velox sp. nov.

Represented by a maxillary, both mandibles, quadrate, numerous branchiostegal rays, and a rather long bone, partially concealed by

the rays, which is probably one of the hyoids. The museum number of the specimen is 316.

The maxillary is long and slender and of about equal depth from the superior condyle backward. The condyle is elevated, but how much can not be determined as the superior portion is not preserved. Just beneath the condyle the bone thickens and the outer surface contracts inward. External to the condyle there is a broad shelf of bone which is very roughly striated. The premaxilary surface is not preserved. The alveolar border supports about forty-seven teeth as near as can be estimated. They are conical, directed slightly inward, and closely set. The crowns present a smooth enameled surface. The whole of the posterior portion of the bone is finely striated. Just above the alveolar border in the anterior half of the bone there are many small nutritious foramina leading inward.

The mandible differs from that of *P. latimentum* Cope in not having a tooth on the symphysis within the anterior one, and in having a greater depth at the coronoid with reference to its length.

The dentary is short and strongly incurved at the symphysis. The symphysis is divided by a groove into an external and an internal portion. The external is small and tubercular in its nature. The internal is probably the only part that is in contact with its fellow on the opposite side and it has a well marked ridge extending backward which becomes more indistinct toward the posterior portion. This ridge causes the bone to be thickened just below the alveolar border. The lower portion of the dentary is thin and smooth externally, except on the lower border, where there are short and deep striæ extending backward. The alveolar border supports probably thirty-eight or forty teeth. These are closely set, non-striated and directed inward The external alveolar wall rises considerably above the internal.

The character of the cotyloid cavity can not be made out owing to the quadrate being firmly in place. The outer surface of the articular is covered with striæ which become coarser toward the lower portion.

The head of the quadrate seems to be broad and bifurcated as in *P. leptognathus*. Above the head the bone broadens anteriorly and has a strong ridge extending upward along the posterior border. The bone supposed to be one of the hyoids, has a broad flat bifurcated extremity, which soon contracts and becomes rod like. The other extremity is not preserved. Between the jaws there are several pieces of ossified cartilage covered with minute denticles somewhat resembling shagreen.

MEASUREMENTS.

Maxillary:	length from premaxillary surface	135
"	depth of condyle	26*
Mandible:	length from cotyloid cavity	157
"	length of alveolar border	122
"	depth at coronoid angle	48.3
"	number of teeth in one inch	8
Hyoid (?):	distance across anterior end	23*

In addition to the above the following species have been described from Kansas:

P. kingii Cope. Proc. Am. Phil. Soc., 1872. p. 344.
P. latimentum Cope. l, c., p. 346.
P. sheareri Cope. l. c., p. 347.
P. caninus Cope. l. c., p. 344.
P. leptopsis Cope, Hayden's bull. U. S. Geol. Surv. Terr. No. 2, 1874, p. 42.

BERYX(?) Cuvier.

The material referred to this genus consists of fragments of several individuals, including two species. The bones are different from those of any of the Cretaceous fishes heretofore described from America, but resemble somewhat the figures of *Beryx* from the chalk of England. For the present I will leave it in this genus.

Beryx polymicrodus sp. nov.

This species is represented by the mandibles, maxillaries, premaxillaries, and other fragments of several individuals.

The maxillary is elongated and slightly concave on the alveolar border, which is rather broad in front and narrow behind, and is covered with numerous villiform teeth directed inward. Just above the most anterior of these there is an elevated articular portion which probably binds the jaw to the skull. The posterior portion is expanded and covered with very coarse striæ.

The premaxillary resembles in a general way some of the recent cat-fishes. The alveolar portion is covered with villiform teeth slightly larger than those upon the maxilla. They are directed inward and increase in size from before backward.

The dentary is rather light in structure and is covered above with teeth similar to those found upon the maxilla and premaxilla. The symphysis consists of an internal and an anterior portion. The first of these is a flat facet, the second a well rounded condyle. The alveolar surface is projected forward over this portion as well as overhanging all the external side of the dentary. Below the alveolar portion the bone is covered with large longitudinal ridges on both the external and internal sides of the bone.

*Estimated.

The cotylus has a small facet for the quadrate, with a prominent hook of bone extending upward and backward on the external side.

Beryx multidentatus sp. nov.

Represented by a fragmentary mandible and a portion of a maxilla. These indicate a fish about one-third larger than the species just described, and differing from this in not having the symphysis divided into two parts, and in having no anterior projection of the alveolar portion over the symphysis. The teeth continue over the external side of the bone instead of overhanging it as in the form just described. The cotylus is larger than in *B. polymicrodus*, concave from above downward and convex from side to side.

Lawrence, Kans., Sept. 29, 1898.

Refractive Index and Alcohol-solvent Power of a Number of Clearing and Mounting Media.

Contributions from the Zoological Laboratory No. 3.

BY C. E. M'CLUNG.

As far as the author's observation goes there has not been published any very extensive list of the refractive index and clearing value of the reagents commonly used by microscopists. Such a want is noted by Lee in the fourth edition of his "Vade Mecum," page 66. In the hope of assisting in the compilation of such valuable information, the writer has experimented on the reagents named in the following table with the results there noted. The instrument employed in the determination of the refractive index was the Pulfrich refractometer, the sodium flame being used as the source of illumination and the conditions of temperature being kept as nearly constant as possible.

In ascertaining the clearing value of the substances a number of methods were tried but finally abandoned for the simple one of testing the strength of alcohol that would dissolve in the reagent and produce a clear solution. While this method does not truly represent the conditions present in a tissue where the clearing agent is replacing the absorbed alcohol, yet actual practice indicates that the results are approximately correct, at least nearly enough so to make the figures of practical value. Strengths of alcohol varying by 5 per cent. were employed, and the weakest one that the reagent would dissolve is given in the table as the lowest grade of alcohol from which it will clear.

The different substances tested were taken from the ordinary laboratory supply and supplemented by others obtained from the Pharmacy department of the University. The latter consisted almost entirely of essential oils from the house of Lehn & Fink, New York. When possible, more than one sample of each reagent was obtained and tested.

Following is the table:

	Refractive Index.	Strength of Alcohol dissolved.	Price per pound.
Chloroform	1.4395	95 per cent.	$ .65
Linalool	1.45941	80 "	....
Oil Eucalyptus	1.46090	90 "	1.25
Linaloe	1.46090	85 "	....
Oil Petitgrains	1.46090	90 "	....
" Coriander	1.46288	85 "	.75 oz.
" Peppermint	1.46327	85 "	1.25
" Cedar	1.46626	95 "	.30
" Pinus sylvestris	1.46882	100 "	1.10
Turpentine	1.46882	100 "	1.40 gal.
Oil Lemon	1.47078	95 "	1.25
" Eucalyptus glob.	1.47097	90 "	....
" Orange	1.47176	95 "	4.00
" Pinus picea	1.47274	100 "	....
" Juniper berries	1.47394	95 "	1.75
" Pinus pimlionis	1.47620	100 "	....
" Citronella	1.47909	90 "	1.25
" Origanum	1.47919	95 "	1.10
Turpineol	1.48005	75 "	....
Oil Celery	1.48054	95 "	1.20
" Nutmeg	1.48054	95 "	.30 oz.
" Origanum	1.48103	95 "	1.10
" Caraway	1.48441	90 "	1.75
" Ginger	1.48807	95 "	.65 oz.
Xylene	1.49348	95 "	....
Benzene	1.49488	95 "	.45
Carvol	1.49535	85 "	....
Oil Thyme	1.49638	95 "	1.49
" Copaiva	1.49723	Immisc.	1.10
" Cedarwood	1.50188	95 per cent.	....
" "	1.50326	95 "	....
" Cumin	1.50373	90 "	4.50
" Sandalwood (E. In.)	1.50510	90 "	6.00
" Calamus	1.50602	95 "	.30 oz.
" Sandalwood (W. In.)	1.50820	80 "	3.50
" Cedarwood	1.51533	95 "	....
Xylene Balsam	1.52397	.. "	....
Oil Sassafras	1.52724	95 "	.50
" Allspice	1.53062	85 "	.25 oz.

	Refractive Index.	Strength of Alcohol dissolved.		Price per pound.
Oil Cloves	1.53171	80	"	.65
" Sweet Birch	1.53329	90	"	....
" Cinnamon leaves	1.53535	85	"	....
Safrol (sp. gr. 1.108)	1.53584	95	"	....
Oil Fennel	1.53885	95	"	1.75
" Cloves	1.53723	80	"	.65
" Mirbane*	1.54982	95	"	.35
" Anise	1.55795	95	"	2.25
Anethol	1.55208	90	"	....
Anilin	1.58457	60	"	....
Oil Cassia	1.60160	80	"	1.80

The table speaks for itself, so that comment is scarcely necessary. From the list given, a series of clearing agents may be selected in which almost any refractive index from 1.44 up to 1.60 is obtainable. Since many of these are suitable agents in which to mount objects, the proper refractive power for most structures may be selected. Where it could be found, the price of the reagent per pound is given.

Particular attention is called to the value of the oil of cassia, which has a refractive index of 1.60160 and clears from 80 per cent. alcohol. It is a most excellent reagent according to all experience in the laboratory, and in addition to its other good qualities it dries hard enough to make permanent mounts.

On Some Turtle Remains from the Ft. Pierre.

Contributions from the Paleontological Laboratory, No. 37.

BY GEORGE WAGNER.

Among the material collected this summer (1898) by the University Geological Expedition are some remains of turtles, from the Ft. Pierre. Though fragmentary, they are worthy of study; firstly, because they seem to be the first material of this nature collected from this horizon in Kansas, and almost the first from any locality; and secondly, because of their remarkable relation to the turtles described from the underlying Niobrara.

Toxochelys latiremis Cope.

A specimen consisting chiefly of the back part of the skull and a part of the upper jaw must evidently be referred to this species. The material (Kas. Univ. Museum, No. 1221) is in the poor state of preservation so common with Ft. Pierre fossils; it shows, however, fairly well most of the quadrate, the basisphenoid, and parts of the pterygoid, maxillary, basi and supra-occipital, and parietal. The sutures are indistinct except between the quadrate and pterygoid. (See fig. 1.)

On comparison of these elements with the corresponding parts of two skull of *T. latiremis* collected from the Niobrara, (Kans. Univ. Mus. Nos. 1214 and 1215),* I fail to find any difference in specific characters; even in size they are nearly identical. A difference in the appearance of the articular surface of the quadrate is undoubtedly due to difference in compression. This specimen was discovered by Professor Williston at Eagle Tail Creek, near Sharon Springs, Kas.

Another specimen (Kansas University Museum 1222) consists of fragments of a lower jaw, including one nearly complete

*See University Geol. Survey of Kas., Vol. 4, Plate 79.

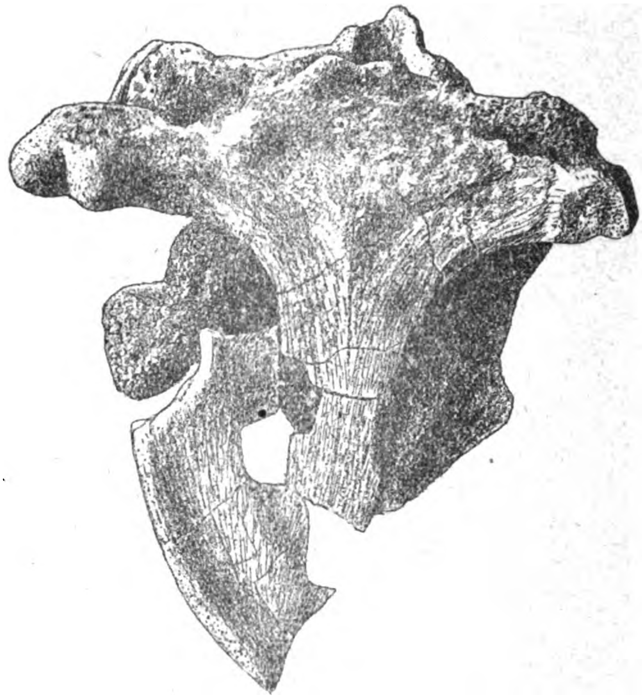


Fig 1.

ramus. The sutures can be traced fairly well and the jaw is but little distorted (See fig. 2.). In the interrelation of the bones and in the slender symphysis this ramus greatly resembles that of *T. latiremis*, but the ramus as a whole is very much stouter than that of *latiremis*. Its length is about twice that of the only *latiremis* jaw in the University museum (No. 1215), and even exceeds that of

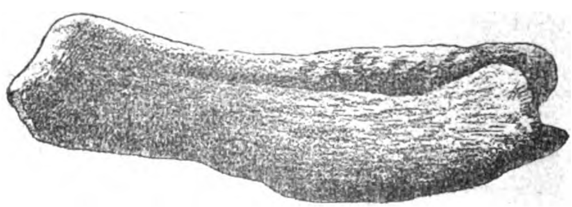


Fig. 2.

a specimen figured by Cope.* The specimen was discovered by Mr. Sydney Prentice near Lisbon, Wallace county, Kas.

The discovery of the same species of vertebrates in the Ft. Pierre and Niobrara, of which the first mentioned above is one of

*Cretaceous Vertebrates, Plate VIII, Figs. 1, 1a.

several examples, is certainly worthy of note, and throws doubt upon the propriety of the terms *Colorado* and *Montana* in Cretaceous stratigraphy. That the formation whence the specimen came is Ft. Pierre is evident from the invertebrate fauna. Prof. Williston also informs me that the genus *Platecarpus*, and possibly the species *coryphæus*, occur in the same horizon. Furthermore Prof. Case considers* Mr. Wieland's *Archelon ischyros*,† from the Ft. Pierre of South Dakota, congeneric, if not conspecific, with the *Protostega* of the Niobrara.

Prof. S. W. Williston has furnished me with the above specimens, as well as with constant aid and encouragement in this and other work. To him my deepest gratitude is due.

*Journal of Morphology, Vol. 14, P. 52, (1897).
†Am. Jour. Sc. Vol. II, Dec., 1896. P. 399, 1 Plate.

Parasitic Influences on Melanoplus.

Contribution from the Entomological Laboratory. No. 63,

BY S. J. HUNTER.

The relations existing between the host and its parasite is an ever interesting source of study from a biologic standpoint. Multiplied numbers of the former tend toward greater increase of the latter. When the parasites predominate, the individuals of the host tribe decrease; should the host disappear, the parasite must follow or adapt itself to new environments. Absence of the parasite grants license to the increase of the host. The prevalence of one is directly dependent upon the other.

In order that an estimate of the influence of this condition upon *Menaloplus differentialis* might be gained, the writer while conducting the summer field work of this department during the two seasons past, collected a number of the differential locust. Fifty were taken the first week of October, 1897, one hundred and thirty were collected September 3, 1898. 12 per cent. of those captured in 1897 had been parasitized by diptera. Of those taken in 1898, 20 per cent. had been attacked by parasite diptera. When it is taken into consideration that the dates of capture were before the close of the active season of the the parasites, and that by reason of capture and confinement, some of the locusts taken were doubtless saved from attack, the estimate can be regarded as conservative. The duration of observation and number of individuals considered will not yet allow favorable deductions to be made from the 8 per cent. increase recorded this year. In localities where this locust was superabundant in October, 1897, the number of dead forms showing an unmistakable evidence of the work of dipterous parasites was nearly equal to those moving about. The number of *Melanoplus differentialis* that appeared in the same localities this season was equal to, if not greater than, those existing the year previous. This species of locust has been of economic importance annually in those regions for some years past. This

is in a measure due to the peculiarly favorable condition existing there, environments which appear to be highly suitable to the rapid multiplication of this species. The ultimate effect of parasitism upon *Melanoplus differentialis* with such surroundings is yet to be demonstrated.

Observations on this subject will be continued by the department. It is the purpose of this paper to record the data observed and diptera concerned. Dr. S. W. Williston and Dr. Garry de N. Hough have very kindly examined the specimens bred. The descriptions and determinations of the Sarcophagidæ made by Dr. Hough appear below. The description of the Tachinidæ which appear to be new, will shortly be given by Dr. Williston in a paper on the museum types of Tachinidæ.

Concerning the life history of the diptera described in this article, the following notes have been made. Careful and continued watching for the act and time of oviposition was not fully rewarded. During the period of the last moult of *Melanoplus differentialis*, when frequently a dozen individuals could be seen at one time in various stages of this change, the writer noted numbers of Sarcophagidæ flitting nervously over and about, alighting near the soft viscid locust, then taking wing again. While no act of oviposition or darting downward was observed, as is the case with many parasitic Hymenoptera when placing their eggs, it is the writer's opinion that some at least of the eggs are placed upon the locust at this time. This belief is strengthened by the fact that the insect, during the moult, is quiescent, is soft and lightly coated with a sebaceous fluid and therefore is an easier prey and a greater attraction for parasitic flies in quest of a host than the active and fully chitinized insect.

The lot of specimens from which *Sarcophaga cimbicis* was bred was collected on September 30, 1897; the larva came forth from the host four days later. It emerged October 23, 1897. The material from which *Sarcophaga hunteri* was bred was taken on September 1, 1898. Three of these dipterous larvæ pupated on the 3d, one on the 6th, and the last of the five on the 9th of September. They emerged in the following order: two on September 6, one on September 8, the remaining two, a male and a female, now in Dr. Hough's collection, hold the labels giving date of emergence, a copy of which I did not retain. There elapsed, however, in each case but a few days between pupation and maturation.

Sarcophaga cimbicis Town. Can, Ent. Vol. 24, pp. 126, 127, 1892.

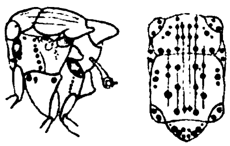
This specimen, a female, was determined by Dr. Hough from

material in his collection. He wrote that the description by Townsend was not then accessible. Upon looking up the list of types in our collection, I found the material, a male and a female, upon which Townsend based his description. A careful comparison with literature at hand, made by Dr. Williston, shows the three specimens to be without doubt identical It is interesting to note, as showing the range of adaptability of this species, that the types were bred from cocoons of *Cimbex Americana.*

Here follows Dr. Hough's description:

Sarcophaga (Tephromyia) hunteri nov. sp. Three males and two females, bred from *Melanoplus differentialis* by Mr. S. J. Hunter in whose honor I have named it. Habitat Kansas.

Length five and one-half to seven millimetres. Color gray; the male rather brownish, the female whitish. Abdomen without the usual variable spots of a *Sarcophaga* but with three black stripes, a median and on each side a lateral; in the female the lateral stripes are quite faint and can only be seen well with a favorable incidence of light. Anal segments gray, retracted within the fourth segment in the males. Palpi yellow to yellowish brown. Antennæ brown with the apex of the second joint and the base of the third yellow to a varying extent. Squamulæ white. Wings grayish hyaline; first longitudinal vein not spinose, third spinose for two-thirds to three-fourths of the distance to the small cross vein. Legs black; in the male more or less brownish gray pollinose, in the female whitish gray pollinose. Hind tibiæ of male not bearded.



Head—Front of male at narrowest point one-sixth the width of the head, from this point which is about at the junction of the dorsal and middle thirds the front widens both dorsad and ventrad. Front of female of uniform width, six-twentieths the width of the head. The exact measurements are: Male front 0.4 mm., head 2 5; female front 0.6 mm, head 2.0 mm.

Antennæ—Third joint more than twice as long as the second. Arista fully as long as the second and third joints together, composed apparently of but two joints of which the basal is very small and about as long as broad, the terminal tapering as usual (its basal and apical thirds black, its middle third whitish) and feathered for rather more than half its length with rather long, fine hairs. The yellow, or perhaps I should say reddish yellow, color is more extensive on the antennæ of the female than of the male.

The vibrissal angle is a little above the mouth edge and slightly but distinctly narrows the cypeus. Dorsad the principal vibrissa the vibrissal ridge is beset with small or minute bristles its entire length. Ventrad the principal vibrissa are about three smaller vibrissæ.

The dorso-ventral diameter of the bucca is one-third that of the eye. It is quite evenly beset with small bristles which are larger toward the edge of the mouth opening where they form a distinct bordering row.

Macrochætæ of vertex, front, etc. Male: By far the largest of the vertical bristles is the inner vertical, the outer vertical is scarcely if at all larger than the cilia of the posterior orbit. The greater ocellar are small, the lesser ocellar very small; of the latter there are several pairs and they extend over upon the occipital surface of the head beyond the postvertical pair which is small and very evidently a member of the ocellar group. The occipito-central is present and is about as large as the postvertical. There are two or three ascending and about eight decussating transverse frontals. The latter extend down upon the gena about as far as the apex of the second antennal joint. Upon the geno plate laterad the frontals there are no large bristles but an irregular row of exceedingly minute hairs which begins at or a little dorsad of the middle of the geno-vertical plate and extends ventrad on the geno-vertical plate and on the gena nearly or quite to the ventral end of the latter. On the gena this row has a tendency to become double and the last three to five members of the anterior row are much larger than the rest, thus forming a rather prominent little group near the lower corner of the eye. The ciliæ of the posterior orbit are small, closely set and well aligned. Parallel to them is a second distinct row of bristles of about the same size.

Female: The bristles of the head of the female differ from those of the male as follows. The outer vertical is almost as wel ldeveloped as the inner vertical. The transverse frontals number but five or six. The row of minute hairs on the geno-vertical plate and gena has a lesser tendency to become doubled on the gena. Two good sized orbital bristles are present.

Thorax—The thorax is striped as is usual in *Sarcophagæ* The stripes are very distinct in the male and quite faint in the female. The chætotaxy of the thorax is alike in the two sexes and is indicated in the accompanying diagram. The female has a smaller number of minute bristles than the male and consequently its chætotaxy is more easily made out. In the diagram I have indicated three post

humeral bristles. The two smaller ones are in but one specimen large enough to be distinguished from the other hairs or microchætæ. This variation of the posthumerals is common in *Sarcophagæ*.

Abdomen—The macrochætæ of the abdomen are marginal only. Each segment has a complete row. On the first and second segments they are all of insignificant size except two or three at the lateral border. On the third segment all are of good size and they number twelve to fourteen. On the fourth segment all are of good size ond they number fourteen to sixteen.

The bristles of the legs are arranged as is usual in *Sarcophagæ* I can make out nothing worthy of especial notice here.

Wing—First longitudinal vein not spinose. Third vein not spinose for two-thirds to three-quarters of the distance to the small cross vein. Elbow of fourth almost exactly rectangular and provided with an apparent appendix which, however, is not a stump of a vein but a slight fold or wrinkle of the wing. Hind cross vein sinuous, longer than, but hardly twice as long as, that segment of the fourth vein between it and the elbow. Hind cross vein and apical cross vein almost exactly parallel.

This species belongs to Brauer's subgenus *Tephromyia* of *Sarcophaga* (sens. lat.). In this subgenus the vibrissal angles are distinctly above the mouth edge and, projecting somewhat mesad, distinctly narrow the clypeus. The abdomen does not have the changeable spots, maculæ spuriæ, of *Sarcophaga* but is either unicolorous or marked with fixed spots or lines. The European species of this group are *T. grisea* Meig., *T. lineata* Fall., *T. affinis* Fall., and *T. obsoleta* Fall. As far as I am aware *hunteri* is the first Tephromyia to be observed outside of Europe. Through the kindness of Herr Paul Stein of Genthin, Germany I have now in my possession specimens of *grisea*, *affinis* and *obsoleta*. From these specimens and the accessible descriptions of *lineata* I am able to construct the following table for separating the species of this group,

A—Abdomen unicolorous, squamulæ yellow, wings strongly yellow at base—*grisea* Meig.

AA—Abdomen with distinct black markings, squamulæ not yellow, wings not strongly yellow at base. B.

B—Palpi black. C.

C—Each abdominal segment with a black dorsal line and on each side withe a narrow, oblique, black spot. These spots often

united so that the abdomen presents three black stripes. Front of male one-third the width of the head.—*lineata* Fall.

CC—First abdominal segment blackish, other segments each with a dorsal black line and on each side with a large irregularly shaped black spot. Front of male one-fifth the width of the head; of female one-third the width of the head—*affinis* Fall.

BB—Palpi yellow or brownish yellow.

D—Front of male one-fourth as wide as the head; third antennal joint less than one and a half times as long as the second; no intraalar bristle in front of the suture—*obsoleta* Fall.

DD—Front of male one-sixth, of female less than a third as wide as the head; third antennal joint more than twice as long as second; with an intraalar bristle in front of suture—*hunteri* n. sp.

A Graphical Method for Constructing the Catenary.

BY WALTER K. PALMER.

INTRODUCTION.

An inspection of the various works on mathematics, mechanics, graphics and drafting will reveal the fact that, while ready drawing-board constructions are available for most of the curves encountered in engineering drafting, no method has been presented for constructing the catenary which is applicable to all the sets of conditions that may from physical considerations be imposed. Many discussions of the curve are offered; but, with no exception thus far noted, they either deal with one or two simple cases of construction, neglecting wholly the more general cases, or presume some kind of an approximation or involve a series of tedious computations which, in the end, yield only an approximate result; and in most instances these objections are all present. No purely graphical construction, such as is desirable from the standpoint of the draftsman, has been given, it is believed, even for the mere plotting of the curve when it is not required to conform to fixed conditions.

While this curve is, perhaps, not of such general importance or wide application in engineering drafting as some others for which ready constructions are available, it would still seem that a direct and exact drawing-board construction for it, such as the constructions for the parabola, hyperbola, etc., would be of interest and value to draftsmen generally.

An examination of the equation of the catenary would seem at first to show conclusively that the objections mentioned are entirely unavoidable. It would appear that no means can be had for determining the curve in conformity to given conditions, which does not involve approximations of some sort, since the parameter of the catenary is involved in a transcendental equation, impossible to solve by ordinary algebraic methods; and the form of this equa-

tion is not such as to suggest at once a graphical method for plotting even the simplest case of the curve.

But a closer study of the equation makes possible a method not only suitable for drawing the simple cases of the curve, but likewise for plotting a catenary to conform to any set of conditions which may have a physical realization, with exactness, and with no computing whatever.

To make clear the meaning of all features of this equation, and to lead logically to the demonstration of the rules which are to be presented for the various cases, the equation of the catenary will be deduced and some of its most notable properties considered with their graphical illustrations.

EQUATION OF THE CATENARY.

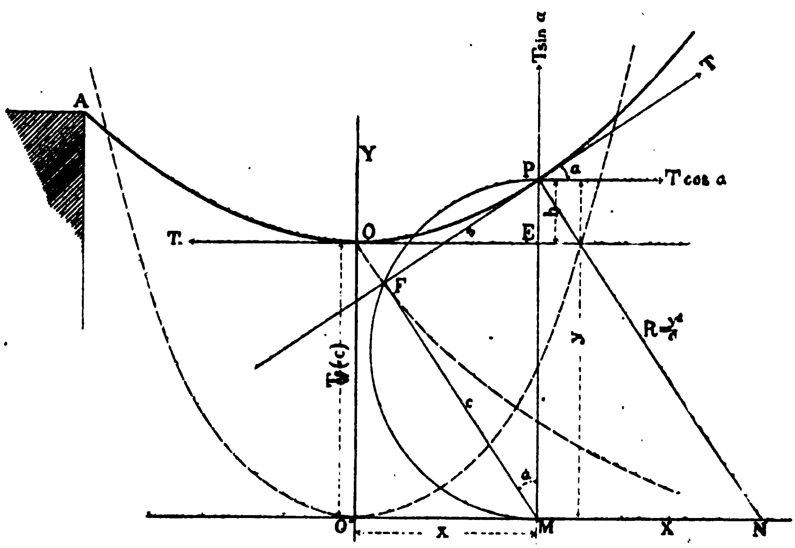


Fig. 1.

Fig. 1 shows the cord of uniform weight suspended from a support, A, falling to a lowest point, O, called the vertex, and rising to another support omitted from the figure.

Take for the origin of coördinates the lowest point, O. Now consider a portion of the cord, as it hangs in equilibrium, between the lowest point O and any point P, the coördinates of which, with reference to O, are x and y.

Let T be the tension in the cord at this point P, and T_0 the tension at the lowest point O.

Letting also

w=weight of cord, per unit of length,

W=total weight of cord from O to any point P, and

s=the length of the cord from O to P,

we have the following relations regarding the piece of cord as a body in equilibrium:

$$\left.\begin{aligned} T_0&=T\cos\alpha \\ W&=T\sin\alpha \\ W&=ws. \end{aligned}\right\} \qquad (1)$$

From which, by dividing,

$$\frac{T\sin\alpha}{T\cos\alpha}=\tan\alpha=\frac{dy}{dx}=\frac{W}{T_0}=\frac{ws}{T_0}. \qquad (2)$$

$$\therefore\quad \frac{dy}{dx}=\frac{w}{T_0}\int_0^x\left[\left(\frac{dy}{dx}\right)^2+1\right]^{\frac{1}{2}}dx.$$

Differentiating, we have

$$\frac{d\left(\frac{dy}{dx}\right)}{\left[\left(\frac{dy}{dx}\right)^2+1\right]^{\frac{1}{2}}}=\frac{w}{T_0}dx,$$

which integrates to

$$\frac{w}{T_0}x=\log\left\{\left(\frac{dy}{dx}\right)+\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right\}+C.$$

But, since when $x=0$, $\frac{dy}{dx}=0$, we have $C=0$, and

$$\therefore\quad \frac{dy}{dx}+\sqrt{1+\left(\frac{dy}{dx}\right)^2}=e^{-\frac{w}{T_0}x}.$$

Transposing $\left(\frac{dy}{dx}\right)$ and squaring,

$$1=e^{\frac{2wx}{T_0}}-2e^{\frac{wx}{T_0}}\left(\frac{dy}{dx}\right).$$

From which

$$\left(\frac{dy}{dx}\right)=\frac{e^{\frac{2wx}{T_0}}-1}{2e^{\frac{wx}{T_0}}}=\frac{1}{2}\left(e^{\frac{w}{T_0}x}-e^{-\frac{w}{T_0}x}\right). \qquad (3)$$

Then, integrating again, we have

$$y=\frac{1}{2}\left(\frac{T_0}{w}\right)\left(e^{\frac{w}{T_0}x}+e^{-\frac{w}{T_0}x}\right)-\left(\frac{T_0}{w}\right). \qquad (4)$$

which is the general equation of the Catenary, referred to O as the origin.

By shifting the origin to O″, a distance $\frac{T_0}{w}$ below O, the constant of integration disappears, and we have the form

$$y=\frac{1}{2}\cdot\frac{T_0}{w}\left(e^{\frac{w}{T_0}x}+e^{-\frac{w}{T_0}x}\right). \tag{5}$$

For convenience, represent $\frac{T_0}{w}$ in the equation by c. Then

$$y=\frac{c}{2}\left(e^{\frac{x}{c}}+e^{-\frac{x}{c}}\right), \tag{6}$$

which is the usual form for the equation.

ANALYTICAL PROPERTIES OF THE CURVE.

Combining equations (2) and (3) we have

$$\frac{dy}{dx}=\frac{w}{T_0}\cdot s=\frac{1}{2}\left(e^{\frac{w}{T_0}x}-e^{-\frac{w}{T_0}x}\right).$$

$$\therefore\quad s=\frac{1}{2}\,\frac{T_0}{w}\left(e^{\frac{w}{T_0}x}-e^{-\frac{w}{T_0}x}\right),$$

or

$$s=\frac{c}{2}\left(e^{\frac{x}{c}}-e^{-\frac{x}{c}}\right). \tag{7}$$

Squaring (6) and (7) and subtracting,

$$y^2=s^2+c^2; \tag{8}$$

and equation (2) is

$$\tan\alpha=\frac{s}{c}. \tag{9}$$

According to (8) and (9), then, we may draw a tangent at any point on the curve, or find at once, graphically, the angle α, Fig. 1, and also the length of the curve from O to the point, when we have the parameter c known. It is only necessary to draw a semi-circle upon y at the point, and strike an arc with c as a radius from M, Fig. 1, thus forming a right triangle with y, s and c for sides, and determining the tangent at P. Then from the figure, if

$$\left.\begin{aligned} \tan \alpha &= \frac{s}{c} \\ \sin \alpha &= \frac{s}{y} \\ \cos \alpha &= \frac{c}{y} \end{aligned}\right| \qquad (10)$$

From (1) with (10) we have

$$T_0 = T \cos \alpha = \frac{Tc}{y}.$$

And $c = \frac{T_0}{w}$ when the origin was changed,

$$\left.\begin{aligned} \therefore \quad T &= wy \\ T_0 &= wc. \end{aligned}\right\} \qquad (11)$$

This means that the tension at any point in the cord is equal the ordinate of that point, multiplied by the weight per unit length of the cord. From which we see that if a material cord be hung over two smooth pins, as represented in Fig. 2, the position for equilibrium for the cord is such that its extremities are upon the same horizontal line, and that this line is the directrix of the particular catenary formed

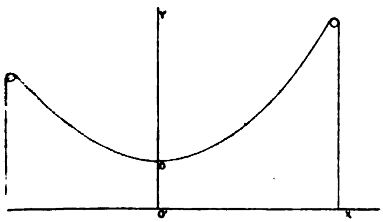


Fig. 2,

From equation (8) and Fig. 1,

$$y^2 = (h+c)^2 = s^2 + c^2.$$

$$\therefore \quad h^2 + 2hc = s^2.$$

$$\therefore \quad c = \frac{s^2 - h^2}{2h}.$$

$$\therefore \quad T_0 = w\left(\frac{s^2 - h^2}{2h}\right). \qquad (12)$$

Radius of Curvature.

$$\rho = \frac{\left\{ 1 + \left(\frac{dy}{dx}\right)^2 \right\}^{\frac{3}{2}}}{\frac{d^2y}{dx}}.$$

From (2)

$$\left(\frac{dy}{dx}\right) = \frac{s}{c};$$

and, differentiating, bearing in mind equations (7) and (6)

$$\frac{d^2y}{dx}=\frac{y}{c^2}. \tag{13}$$

Substituting in the above expression

$$\rho=\frac{\left(\frac{s^2+c^2}{c^2}\right)^{\frac{3}{2}}}{\frac{y}{c^2}}=\left(\frac{y}{c}\right)^3\times\frac{c^2}{y}=\frac{y^2}{c}. \tag{14}$$

This is represented by the length of the normal at point P, between the point and the directrix, that is by $\overline{PN}$, Fig. 1.

For $\quad PN\cos a=y=PN\,.\,\frac{c}{y}\quad$ by the figure and (10).

$$\therefore\quad PN=\frac{y^2}{c}=\rho.$$

Hence the radius of curvature for any point is readily obtained.

Area Under the Curve.

$$A=\int ydx=\int\frac{c}{2}\left(e^{\frac{x}{c}}+e^{-\frac{x}{c}}\right)dx$$

$$=\frac{c^2}{2}\left(e^{\frac{x}{c}}-e^{-\frac{x}{c}}\right)+C$$

$$=cs+(C=0).$$

$$\therefore\quad A=cs. \tag{15}$$

That is, in Fig. 1, the area O″OPM=2× the triangle MFP, or = a rectangle with c and s for sides.

And the area OPE=cs—cx=c(s—x).

Other Properties.

Other properties that may readily be demonstrated directly from the foregoing are:

I. The Involute of the Catenary is the tractrix.

Point F, Fig. 1, is always on the involute of the catenary from the vertex. For FP is constantly equal s, the length of curve OP. FM is constantly tangent to this involute and is of constant length equal c, which is O″O of the figure. The curve, which is the locus of F, is hence the equi-tangential curve, the tractrix.

II. A parabola, with vertex at O″, Fig. 1, and focus at O, when rolled on O″X will generate a catenary, the focus O tracing the curve.

Proof:

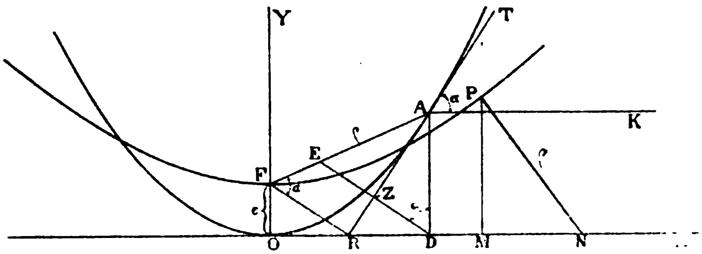


Fig. 3.

Let OA, Fig. 3, be a parabola with focus at F. If the focus of the parabola traces the catenary FP as the parabola rolls on its tangent ON, at O, it must be that when F is at any point P on the catenary, arc OA of the parabola must equal ON, FA must equal the radius of curvature of the catenary at P, PN; and FR, the perpendicular from F upon the tangent to the parabola at A must equal PM, or y of the catenary.

Call the coördinates of A on the parabola (x', y') and those of the point P on the catenary (x, y). Call the radius of curvature of the catenary ρ, and the angle the tangent makes at A, α.

Now, from the properties of the parabola, RA bisects $\angle$ FAD, AE=AD, EF=c, R is always on ON, and $\angle$ RFA= $\angle$ KAT=α.

Then from the figure $$\rho = y' + c \qquad (16)$$

$$\tan\alpha = \frac{dy'}{dx'} = \frac{\sqrt{\rho^2 - y^2}}{y}$$

and

$$\frac{dy'}{dx'} \text{ for the parabola } x^2 = 4cy \text{ is } \sqrt{\frac{y'}{c}}.$$

$$\therefore \quad \frac{y'}{c} = \frac{\rho^2 - y^2}{y^2}. \qquad (17)$$

Eliminating y' with (16) and (17)

$$\frac{\rho - c}{c} = \frac{\rho^2 - y^2}{y^2},$$

which gives

$$\rho = \frac{y^2}{c}.$$

Equation (14) for the catenary gives the radius of curvature ρ equal $\frac{y^2}{c}$. Therefore the curve traced by the focus of the parabola is the catenary.

III. The center of gravity of the catenary is lower than that of any other curve which can be formed from the same length of line between the two fixed points in space. If this were not so the cord would of its own weight fall in to the form having a lower center of gravity.

The center of gravity for any catenary may be located at once, very easily, thus; Fig. 1.

Bisect ON. From this middle point of ON draw a parallel to PN. When this line cuts OY is the center of gravity,

For by the usual method of mechanics the height of the center of gravity of the catenary above the X axis is

$$\bar{y}=\frac{1}{2}\left(y_1+\frac{cx_1}{s_1}\right),$$

and by similar triangles it can easily be shown that NP produced, Fig. 1, cuts OY produced in a point which is at a vertical distance above y_1, or point P, equal to $\frac{cx_1}{s_1}$.

PLOTTING THE CURVE.

CATENARY THE SUM OF TWO EXPONENTIAL CURVES.

Analyzing the equation of the catenary

$$y=\frac{c}{2}\left(e^{\frac{x}{c}}+e^{-\frac{x}{c}}\right),$$

it will be noticed that the ordinate y may be considered as the sum of two portions y' and y'', such that

$$y'=\frac{c}{2}e^{\frac{x}{c}}$$

$$y''=\frac{c}{2}e^{-\frac{x}{c}}. \qquad (18)$$

Now $y'=\frac{c}{2}e^{\frac{x}{c}}$ is recognizable at once as the well known exponential or logarithmic curve. And y'' is plainly the ordinate of the same curve, at the same x distance from the vertical axis on the opposite side of the origin.

So if an exponential curve of the form $y=\frac{c}{2}e^{\frac{x}{c}}$ were obtainable, a catenary could be produced from it as in Fig. 4.

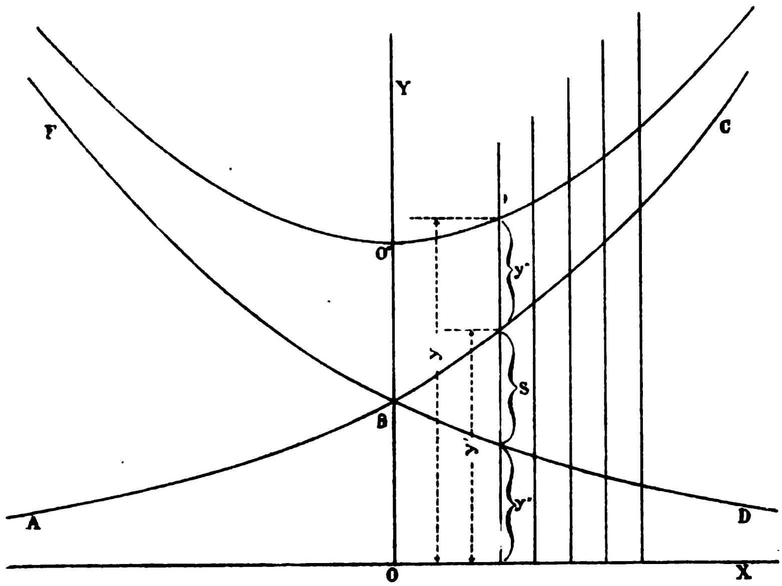


Fig. 4.

It is merely necessary to reverse the curve ABC on the drawing, which can be done readily by transferring points to symmetrical positions by use of the dividers, when the result is the symmetrical curve DBF, whose equation is

$$y=\frac{c}{2}e^{-\frac{x}{c}}.$$

Then, with these two curves plotted in position as shown, a catenary can be quickly obtained by adding ordinates. To do this draw a series of vertical lines conveniently spaced, and with the dividers set off the ordinate of the curve DPF, upward from the curve ABC, on each vertical. Join the points thus plotted with a smooth curve and the result is the catenary.

It will be noticed, also, that the value of s, the length of the catenary, is at once shown at every point along the curve by this figure. For, by equation (7),

$$s=\frac{c}{2}\left(e^{\frac{x}{c}}-e^{-\frac{x}{c}}\right).$$

That is, s is the difference of ordinates at each point, as shown in Fig. 4. These considerations show then that if a satisfactory drawing-board construction can be found for this exponential curve a means is already at hand for plotting the catenary, at least for the simple case of a given parameter.

To discover the desired kind of construction for this exponential curve—one which will admit of the curve being drawn with as much facility as the parabola or ellipse—necessitates leaving the question of the catenary itself for a time to investigate the properties of this exponential curve.

CONSTRUCTION FOR THE EXPONENTIAL CURVE.

The perfectly general form of this curve is

$$y = a\,e^{mx} \qquad (19)$$

Its most notable property is that its sub-tangent is constant.

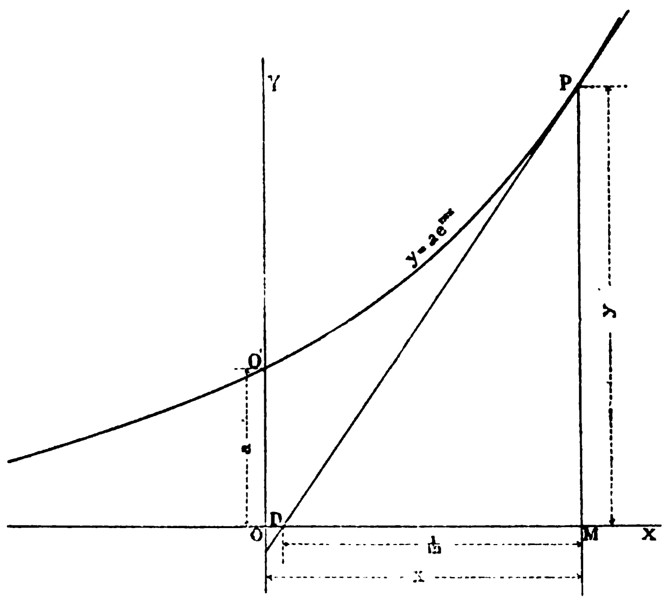


Fig. 5.

That is, DM, Fig. 5, is the same for a given curve wherever the point P may be chosen. For

$$\frac{PM}{DM} = \frac{y}{DM} = \left(\frac{dy}{dx}\right) = a\,m\,e^{mx} = my.$$

$$\therefore \quad DM=\frac{1}{m}. \qquad (20)$$

$OO'=a$, for when $x=o$, $y=a$. If instead of the perfectly general form of the equation we have the form

$$y=\frac{c}{2}\,e^{\frac{x}{c}},$$

which we should have when employing this curve in a construction for the common catenary, then

$$OO'=\frac{c}{2}$$

and the constant sub-tangent $DM=c$.

It would seem now that this fact of a constant sub-tangent should afford the means for the kind of construction desired for this curve. And it is found that the curve may be drawn in a most satisfactory way by its use, as follows:

In Fig. 6 let OY and OX be the axis. Lay off OO′ equal to the constant a and make $OD=\frac{1}{m}$. Or if the form

$$y=\frac{c}{2}e^{\frac{x}{c}}$$

is to be drawn, make

$$OO'=\frac{c}{2},$$

and $$OD=c.$$

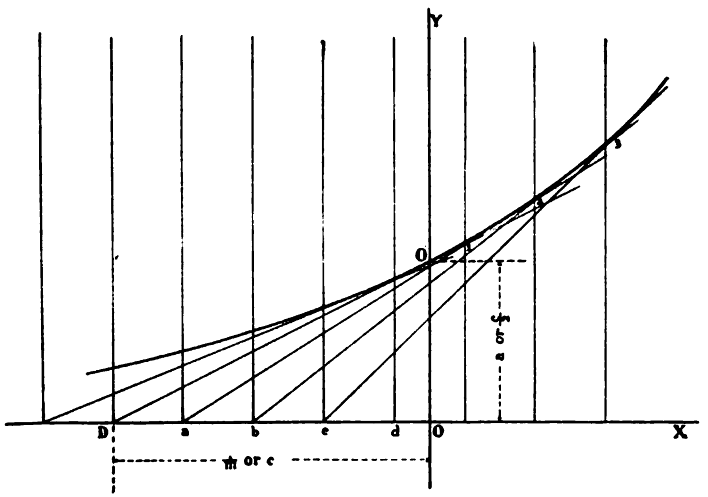


Fig 6

Divide OD into an *odd* number of small equal divisions, say approximately $\frac{1''}{10}$. Then draw vertical lines through every other division point, as shown in the figure, Fig. 6, so that OY will come in the middle of a space between verticals. Draw a straight line through D and O′. Then draw through a and 1, b and 2, c and 3, etc. The resulting series of lines then envelopes the curve, which will be found determined with accuracy without the necessity of drawing a curved line, if the spaces between verticals be made small. The figure will make clear that the curve touching each of these straight lines goes through point O′ and that the condition of a constant sub-tangent equal $\frac{1}{m}$, or c, obtains throughout, and hence that the curve is correctly and satisfactorily drawn.

Having now a simple construction for this exponential curve, and understanding the derivation of the catenary from it, we may easily construct a catenary of any desired parameter; and are, therefore, ready to pass to the consideration of the question of adapting this method to more general cases of the catenary conforming to given sets of conditions as to points, length of cord, etc.

CONSTRUCTIONS FOR THE CATENARY CONFORMING TO GIVEN CONDITIONS.

Aside from the case already noticed of plotting a catenary with a given or assumed parameter, we have from purely physical considerations the following important cases:

CASE I.—SUPPORTS THE SAME HEIGHT GIVEN.

(a) Given a certain length of cord greater than the distance between supports, required the level to which the curve will fall and a plotting of the catenary formed.

(b) Given the level to which the curve is to fall, required the length of cord and a plotting of the catenary.

CASE II.—SUPPORTS AT DIFFERENT LEVELS.

(a) Same as under I.

(b) Same as under I.

Remarks: It should be noticed that the weight per unit length of the cord does not affect the form of the curve, but does determine the tension in the cord.

The tension of the cord on the abutment or at any point, total weight of cord, etc., can, of course, all be readily determined graphically for each of the cases above mentioned, when once the curve is plotted, by means of the properties already considered, so these features will not be treated again in the discussion of these cases.

Discussion of the Cases.

Preliminary to undertaking the treatment of these four cases, prepare as permanent diagrams which will serve for the solution of all problems that may be met so long as the same system of units is used, the following auxiliary drawings:

(1) On one sheet an accurate plotting of a catenary with the constant equal one, that is with equation

$$y = \frac{1}{2}\left(e^{x} + e^{-x}\right).$$

This may be called the "simple catenary."

(2) On another sheet plot from the same construction work which gave the simple catenary a curve of values of s of this curve. That is obtain a plotting of

$$s = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

by taking the difference of ordinates, Fig. 4, instead of the sum, as already explained.

The one-half of each of these curves is sufficient. By means of these two plottings any problem in connection with the construction of a catenary may now be treated.

Constructions for the Four Cases.

CASE I.—SUPPORTS OF THE SAME HEIGHT.

(*a*) *Having given a length of cord and fixed abutments to plot the curve of the cord.*

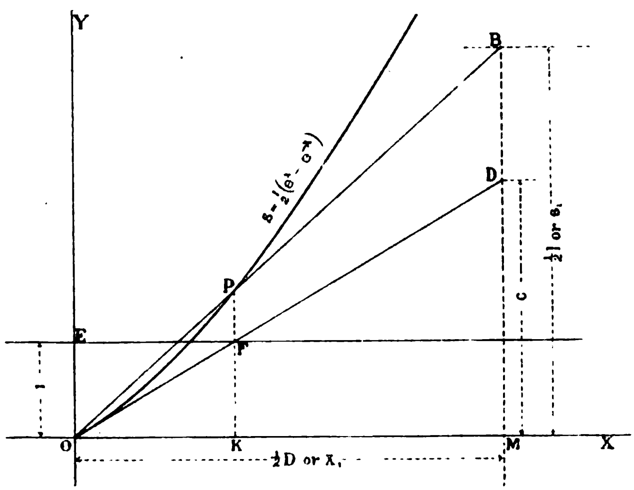


Fig. 7.

For this take the "s-curve" plotting, already prepared, as shown in Fig. 7.

Lay off one-half the distance D between the given abutments along OX and $\frac{1}{2}l$, the length of cord given, upward from M to B.

$$\frac{1}{2}D=x_1$$

$$\frac{1}{2}l=s_1.$$

Join B to O and from P, where BO cuts the s-curve drop a perpendicular PK upon OX.

Draw EF parallel to OM at unit's distance above it. Draw OF to D, when DM is the value of c, the parameter to be used in plotting the required catenary. With this value for c the curve will have the given length between the fixed abutments at given distance, D, apart.

It only remains to lay off c downward from O on another drawing and proceed to construct the catenary by the method already explained.

Proof: The required catenary will, we know from previous considerations, have an equation for s

$$s=\frac{c}{2}\left(e^{\frac{x}{c}}-e^{-\frac{x}{c}}\right).$$

It will be of such proportions that if plotted on Fig. 7 OM and MB would be coördinates of it, when O is the origin. Manifestly we can not substitute

$$OM=x_1 \text{ and } MB=s_1,$$

for x and s, respectively, and solve for c, as we should do in the case of an equation of ordinary kind, owing to form of this equation, but the constant c must be determined in order to plot the catenary. And it may now be readily seen that the graphical steps just indicated do give c correctly.

By the equation of the simple s-curve

$$PK=\frac{1}{2}\left(e^{\overline{OK}}-e^{-\overline{OK}}\right).$$

Now $\qquad PK\times$ some constant $c=BM$

and $\qquad OK\times$ the same constant $c=OM$

by similar triangles.

Substituting in the above equation we have

$$\overline{BM} = \frac{c}{2}\left(e^{\frac{\overline{OM}}{c}} - e^{\frac{OM}{c}}\right).$$

That is

$$s_1 = \frac{c}{2}\left(e^{\frac{x_1}{c}} - e^{-\frac{x_1}{c}}\right).$$

which is just the equation of our s-curve for the required catenary when passing through the given point. And c is simply the ratio of BM to PK or OM to OK, which ratio is easily obtainable by the use of the unit's line as explained. Since FK is one unit DM is the ratio OM : OK.

With the curve plotted, the level to which it drops is easily measured at once, and all other facts in regard to it readily derived.

(*b*) *Having given fixed abutments and the level to which the curve is to drop, to find the length of cord required and to plot the catenary.*

Here instead of the s-curve use the drawing of the "simple catenary." Proceed just as in the first case, and find c, when the curve can be plotted at once and s found, and all the other features as in Fig. 1.

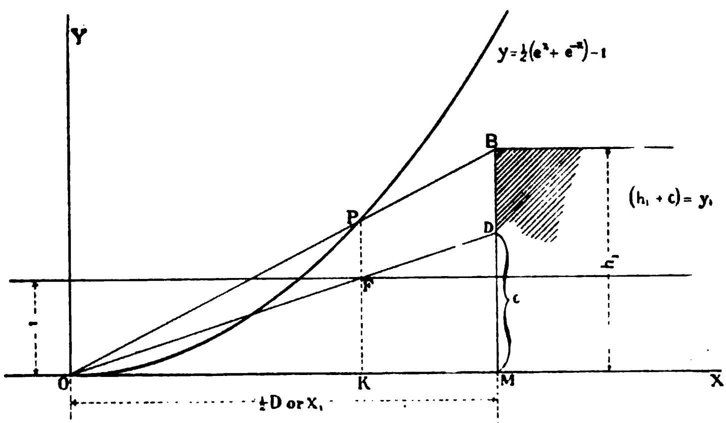


Fig. 8.

Fig 8 shows fully the steps taken on the sheet of the simple catenary. These are precisely the same as in the case first considered.

Proof: For here we know that the required catenary, since it must go through the point of support B, will have an equation such that

$$h_1 = \frac{c}{2}\left(e^{\frac{x_1}{c}} + e^{-\frac{x_1}{c}}\right) - c.$$

And from the simple catenary we have

$$PK = \frac{1}{2}\left(e^{\overline{OK}} + e^{-OK}\right) - 1.$$

But, as before

$$PK = \frac{BM}{c}$$

$$OK = \frac{OM}{c}.$$

Then substituting

$$\frac{BM}{c} = \frac{1}{2}\left(e^{\frac{OM}{c}} + e^{-\frac{OK}{c}}\right) - 1.$$

$$\therefore\ h_1 = \frac{c}{2}\left(e^{\frac{x_1}{c}} + e^{-\frac{x_1}{c}}\right) - c,$$

where c is the ratio OM : OK, which is easily found.

CASE II.—SUPPORTS AT UNEQUAL HEIGHTS.

(*a*) *Given the fixed abutments and l, the length of the cord, to find the level to which the curve will fall and to plot the catenary.*

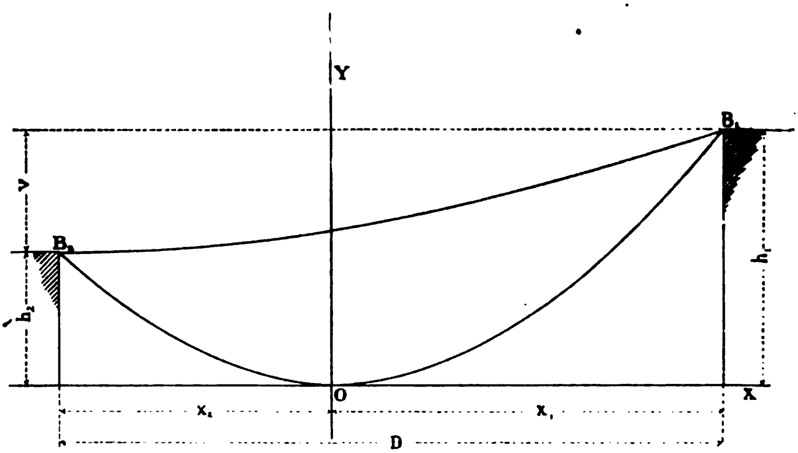


Fig. 9.

Fig. 9 shows the abutments as assumed at a distance apart D, and a difference of level v.

For this case we may discover the relation

$$\sqrt{l^2 - v^2} = c\left(e^{\frac{D}{2c}} - e^{-\frac{D}{2c}}\right), \tag{21}$$

in which l, v and D are given quantities, and c is the parameter of the desired catenary. Inspecting this equation we see that it is of just the form of the s-curve, so that c may be obtained from this equation by the same steps as taken in Case I (a), merely noticing that the result derived by these steps will here be twice c, and hence must be divided for the value of c to use in plotting the desired catenary. Simply construct a right triangle to scale, with l for hypotenuse and v for one side. The other side will at once be $\sqrt{l^2 - v^2}$, and should be used precisely as s_1 was in Case I (a). Then D is to be used as x_1 was, when the resulting value will be 2c.

Proof of Equation: Referring to Fig. 9

$$x_1 + x_2 = D.$$

Let

$$x_1 - x_2 = k \text{ (a constant)}$$

$$\therefore \quad x_1 = \frac{D+k}{2}$$

$$x_2 = \frac{D-k}{2},$$

$$l = s_1 + s_2 = \frac{c}{2}\left(e^{\frac{x_1}{c}} - e^{-\frac{x_1}{c}} + e^{\frac{x_2}{c}} - e^{-\frac{x_2}{c}}\right),$$

$$v = h_1 - h_2 = \frac{c}{2}\left(e^{\frac{x_1}{c}} + e^{-\frac{x_1}{c}} - e^{\frac{x_2}{c}} - e^{-\frac{x_2}{c}}\right).$$

Now substitute above values of x_1 and x_2 and obtain

$$l = \frac{c}{2}\left(e^{\frac{D}{2c}} e^{\frac{k}{2c}} - e^{-\frac{D}{2c}} e^{-\frac{k}{2c}} + e^{\frac{D}{2c}} e^{-\frac{k}{2c}} - e^{-\frac{D}{2c}} e^{\frac{k}{2c}}\right),$$

$$v = \frac{c}{2}\left(e^{\frac{D}{2c}} e^{\frac{k}{2c}} + e^{-\frac{D}{2c}} e^{-\frac{k}{2c}} - e^{\frac{D}{2c}} e^{-\frac{k}{2c}} - e^{-\frac{D}{2c}} e^{\frac{k}{2c}}\right).$$

From which

$$l = \frac{c}{2}\left(e^{\frac{D}{2c}} - e^{-\frac{D}{2c}}\right)\left(e^{\frac{k}{2c}} + e^{-\frac{k}{2c}}\right),$$

$$v = \frac{c}{2}\left(e^{\frac{D}{2c}} - e^{-\frac{D}{2c}}\right)\left(e^{\frac{k}{2c}} - e^{-\frac{k}{2c}}\right).$$

Now, squaring, subtracting and extracting the square root

$$\sqrt{l^2 - v^2} = c\left(e^{\frac{D}{2c}} - e^{-\frac{D}{2c}}\right).$$

Case (a) Continued.

It is evident that another series of catenarys can be drawn through the two abutments B_1 and B_2 of Fig. 9 in a different way from those of the class considered. We may have a series of curves of varying parameter through B_1 and B_2, the axis of which are to the left of B_2, and which consequently do not have their lowest points, or vertices, within the limits of the figure.

If analyzed independently it will be found that they may be treated in the same way as those with axis between B_2 and B_1, as would be supposed. They are indeed no different in any way from the other class. Simply, as the cord is made shorter the catenary becomes flatter, removing the axis nearer to the lower abutment, until when a certain length is reached the axis is at the lower abutment. Then with the shortening of the cord the axis moves farther beyond the lower abutment, until finally when the length is too short to reach from one abutment to the other, that is less than the straight line B_2B_1, the construction fails altogether, as it should.

An inspection of Figs. 7 and 9 will show this final limit very satisfactorily. Looking at Fig. 7 it is plain that the line OB will become tangent to the s-curve at some limit as $\angle$ XOB decreases. The limit for XOB, below which the construction fails, can be seen at once by differentiating the s-curve and making x in $\left(\frac{ds}{dx}\right)$ equal zero. This shows that at $x=0$, that is at point O, the angle of the tangent to the curve is 45^0. Now notice that this fixes the limit for the length of the cord just where it would be found from a consideration of the physical conditions of the problem, as shown in Fig. 9.

If $\angle$ XOB is 45^0 at the limit, then there

$$BM=OM.$$

$$\therefore \text{ for this case,}$$

$$\sqrt{l^2-v^2}=D;$$

or

$$l=\sqrt{D^2+v^2}.$$

$\sqrt{D^2+v^2}$ is the length of a straight line from B_2 to B_1, Fig. 9, evidently the extreme minimum limit for the length of cord.

The length of the cord which brings the axis just at the line of the lower abutment can be seen thus:

Substitute in equation of the catenary the values from Fig. 9 and

$$v=\frac{c}{2}\left(e^{\frac{D}{c}}+e^{-\frac{D}{c}}\right)-c. \tag{22}$$

$$= \frac{c}{2}\left(e^{\frac{D}{2c}} - e^{-\frac{D}{2c}}\right)^2$$

This, with equation (21), gives

$$\frac{l^2 - v^2}{c^2} = \frac{2}{c}v.$$

For which

$$l = \sqrt{v(v+2c)}. \qquad (23)$$

(*b*) *The level to which the curve shall fall being given, required the length of cord which will just reach that level, and also the plotting of the curve.*

For this case it appears to be wholly impossible to derive an equation similar to (21) which will serve as a method, as in Case II (a).

But the problem may be solved in a manner entirely satisfactory from the standpoint of the draftsman by resorting to what has not thus far been found necessary—the drawing of a few tentative lines on the "simple catenary" diagram.

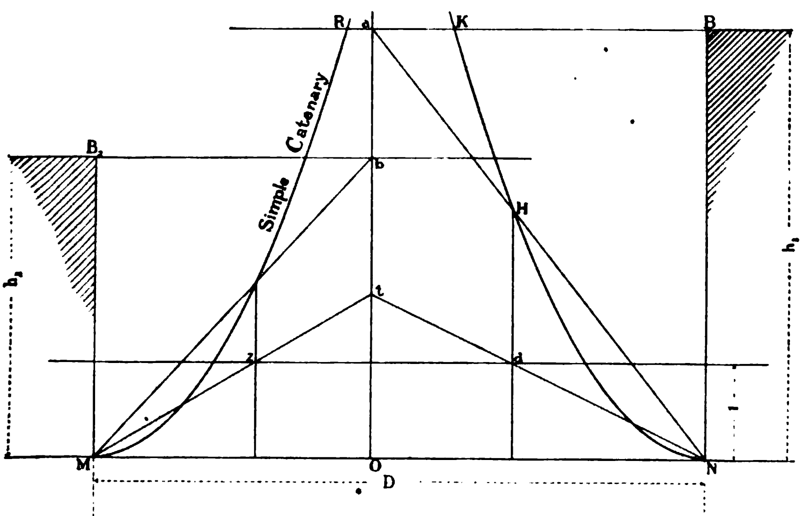


Fig. 10,

Fig. 10, B_1 and B_2 are the given abutments and MN the horizontal line to which the curve is to be tangent.

Lay out this figure on the plotting of the simple catenary already prepared, MFR being the curve. Then transfer a sufficient number of points across by means of horizontal lines and the dividers,

to obtain NHK, the other half of the curve, only plotting so much as will evidently be needed. This can be done very readily.

Now from a study of Fig. 8, and the steps there taken it can be plainly seen that the position for the Y-axis, in the case represented in Fig. 10, must be such as to make possible the diagram of straight lines shown here in this figure.

The portion of the figure on the one side of OY is merely the construction of Fig. 8, applied to the conditions of this half of the curve, and the portion on the other side is the same construction applied to the conditions of that half.

The values determined for c from the two portions of the diagram, must, of course, be the same here, so Mz and Nd must meet OY in the common point t. That is, the vertical axis OY of the required catenary must be so located that lines from a and b, points determined by it, to N and M respectively will give intersecting points, H and F, properly situated, so that the points d and z, dependent upon them, will give Nd and Mz, intersecting at the point t on the axis. Then Ot is the constant c, and may be laid off at once downward from O on OY and the required curve plotted.

It is plain that all of the quantities of this diagram are so involved in each other that the position of OY fulfilling the necessary conditions cannot be located at once definitely. But when the two halves of the simple catenary have been drawn at their proper distance, D, apart, it is possible to see within a very small limit of error just where OY should be for the fulfillment of these conditions. Estimate its position closely thus and draw the diagram, when it will be found that Mz and Nd do not quite meet in the same point on OY. A slight change of the position, now, will bring the two intersections on OY much nearer to coincidence and a very few such trials will locate OY to a degree of accuracy equal to that which could be attained by any method.

Conclusion.

All of the cases which would actually be encountered in the plotting of the curve of a material cord of uniform weight have now been considered and solved entirely by graphical steps. The rules outlined under each case, having been demonstrated, may hereafter be taken as satisfactorily established and a catenary plotted to conform to any set of conditions which may be physically possible, with no computing whatever.

Preliminary Notice on the Correlation of the Meek and Marcou Section at Nebraska City, Nebraska, with the Kansas Coal Measures.*

Contribution from the Paleontological Laboratory, No. 38.

BY J. W. BEEDE.

The appearance of Meek and Hayden's Final Report of the U. S. Geological Survey of Nebraska practically settled the prolonged Meek-Marcou controversy concerning the age of the rocks at Nebraska City, and showed them to be of the typical Upper Coal Measures of the west. Professor Prosser, after an extended study of the Upper Coal Measures and Permian in Kansas, visited Nebraska and studied the relation of the Upper Coal Measures there to those of Kansas, taking the Cottonwood limestone above as a basis. He locates the Cottonwood limestone four miles west of Auburn, Nebraska, about 365 feet above the Missouri river, and refers the Nebraska City rocks, provisionally, to the lower half of the Wabaunsee formation; stating that these rocks have great resemblance to the rocks of that formation as exposed above Topeka, Kansas, on the Kansas river.†

During a short visit at Nebraska City the writer secured nearly all of Meek's species from that locality and Otoe (now Minersville) and the strata of the place were examined as carefully as time would permit. As Professor Prosser had stated, the rocks bore a remarkable resemblance to those near Topeka. Their lithological character, their succession, their fossils and the grouping and preservation of the fossils, bear a striking resemblance to the rocks above and below the Topeka (Osage) coal. At the top of the section there is a calcareous sandstone underlaid by bluish or drab shales,

*For a complete discussion of the subject see the forthcoming Transactions, (vol. 16,) of the Kansas Academy of Science.
†Jour. Geol, Jan.-March, 1897, p. 1, et sec.

which rest upon an argillaceous limestone one to two feet in thickness, which overlies a stratum of very poor coal, or highly carbonaceous shales, beneath which are argillaceous and arenaceous shales and sandstones. The next succeeding strata are covered by the railroad. About thirteen feet above the river (low water mark) there are two thin strata of limestone exposed, beneath which are green and red shales for about four feet.

At Otoe (now Minersville) there are over a hundred feet of variously colored shales and sandstones exposed, which Meek thought to be immediately above the Nebraska City rocks. I was unable to substantiate this by actual observation, but as far as I was able to judge, by looking from the car window and the appearance of the Minersville section, I am inclined to think him correct. He also mentions a stratum of highly carbonaceous shales at one place enclosing a six inch seam of coal in this section. The section has now, I think, practically the same appearance that it had when Meek studied it.

During the past summer the Burlingame limestone was traced from near Topeka to the Nebraska line. Its eastern extension passes northeast from Martin's Hill to near Meriden, where it turns north for about ten miles to the latitude of Valley Falls. East of here it appears in the top of the eminence, on which stands the town of Winchester. From here it trends north nearly to the Nebraska line, but bends westward before crossing the line on account of the valley of the Great Nemaha. It appears, if I have been correct in following it (the disappearance of the escarpments and the great thickness of the drift after entering Brown county, Kansas, and the scarcity of exposures makes it difficult to trace a formation with much certainty) near the base of the bluff on the north side of the Great Nemaha north of the bridge, which is a trifle east of the west line of Irving township in Kansas. Several feet below this, coal is mined at this place.

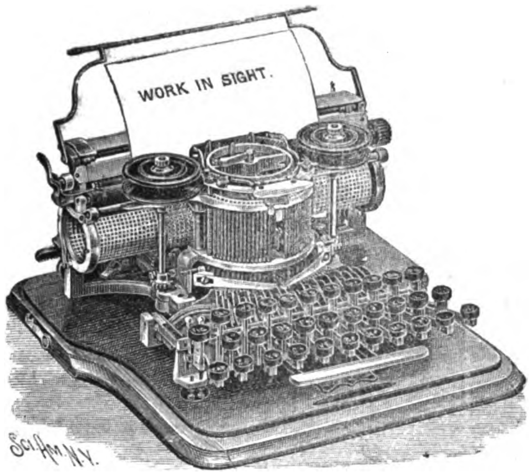
This is the same coal, with the sandstone below, observed by Hayden on the bluffs of the Missouri, at the mouth of the Great Nemaha, a little east of here. Meek was of the opinion, though not certain, that this sandstone was the same as the sandstone seen at Peru and Brownville, which he places above the Nebraska City section. If this be true it throws the section at Nebraska City in the same general horizon with the Topeka-Osage coal, if it be not identical with it, and the limestone at the base of the section would then represent the Topeka limestone or a part of it. While I have not been over the ground between Minersville and Rulo,

Neb., I am of the opinion that this conclusion is correct. It agrees in the lower part with Prosser's location of the Cottonwood limestone four miles west of Auburn, Neb., and also with his ideas as to the stratigraphic position of the Nebraska City beds.

Editorial Notes.

During the past summer the University Geological Expedition in western Kansas was very fortunate in securing a most extraordinarily good specimen of a *Platecarpus*, which adds, unexpectedly, some new facts in their structure. The specimen was discovered by Mr. A, Stewart a mile and a half from Elkader on the Smoky Hill River, and includes the complete animal to the base of the tail. The skin was preserved entire, but, when exposed to the air, very much of it has flaked off. By the use of shellac, however, considerable patches have been preserved. The scales are similar to those of *Tylosaurus*, but are somewhat larger, and apparently lack a prominent carina. A remarkable peculiarity is the presence of a row of dermal processes along the nucha, from the skull at least as far back as the thoracic region. How much further they may go it is impossible to say, since the bones lie above the posterior end of them. They are about three millimeters in diameter and four or five, perhaps six, inches in length, forming a thick fringe or mane, and resembling very much the thongs along the legs of buckskin trowsers. The sternal apparatus is preserved entire and apparently, like most of the bones, nearly in position. There is a true, bony sternum, of crescentic shape, with a projecting, flattened, spatulate episternum. The paddle shows the outline of the membrane, which joins the body broadly, and has the fifth finger divaricated. There are five carpal bones. Photographic figures of the nuchal fringe, the sternal apparatus and the skin, together with some observations on the food-habits of the animal will be given in the next number of this Quarterly.

S. W. WILLISTON.



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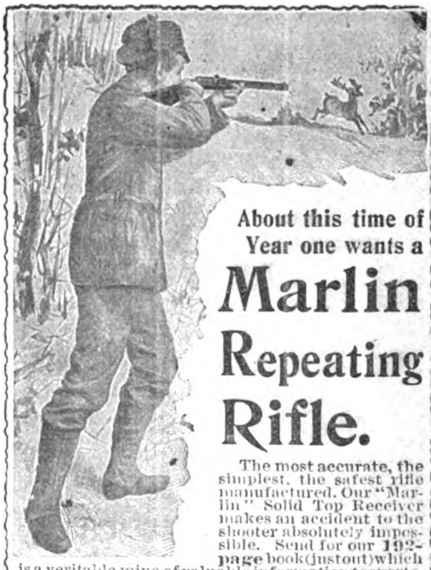

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PLATE I.

Plate I, Figs. 1a, and b, Premaxillary of *Protosphyræna* sp. nov. natural size:

Figs. 2a, and b, Outline and external markings of rostrum of *Protosphyræna bentoniana* Stewart. The first natural size, the second reduced to one-fourth.

Figs. 3a, and b, Maxillary, mandible, and end view of predentary of *Saurocephalus dentatus* Stewart. One-half natural size.

Figs. 4a, and 4b, Internal view of maxillary and mandible of the same.

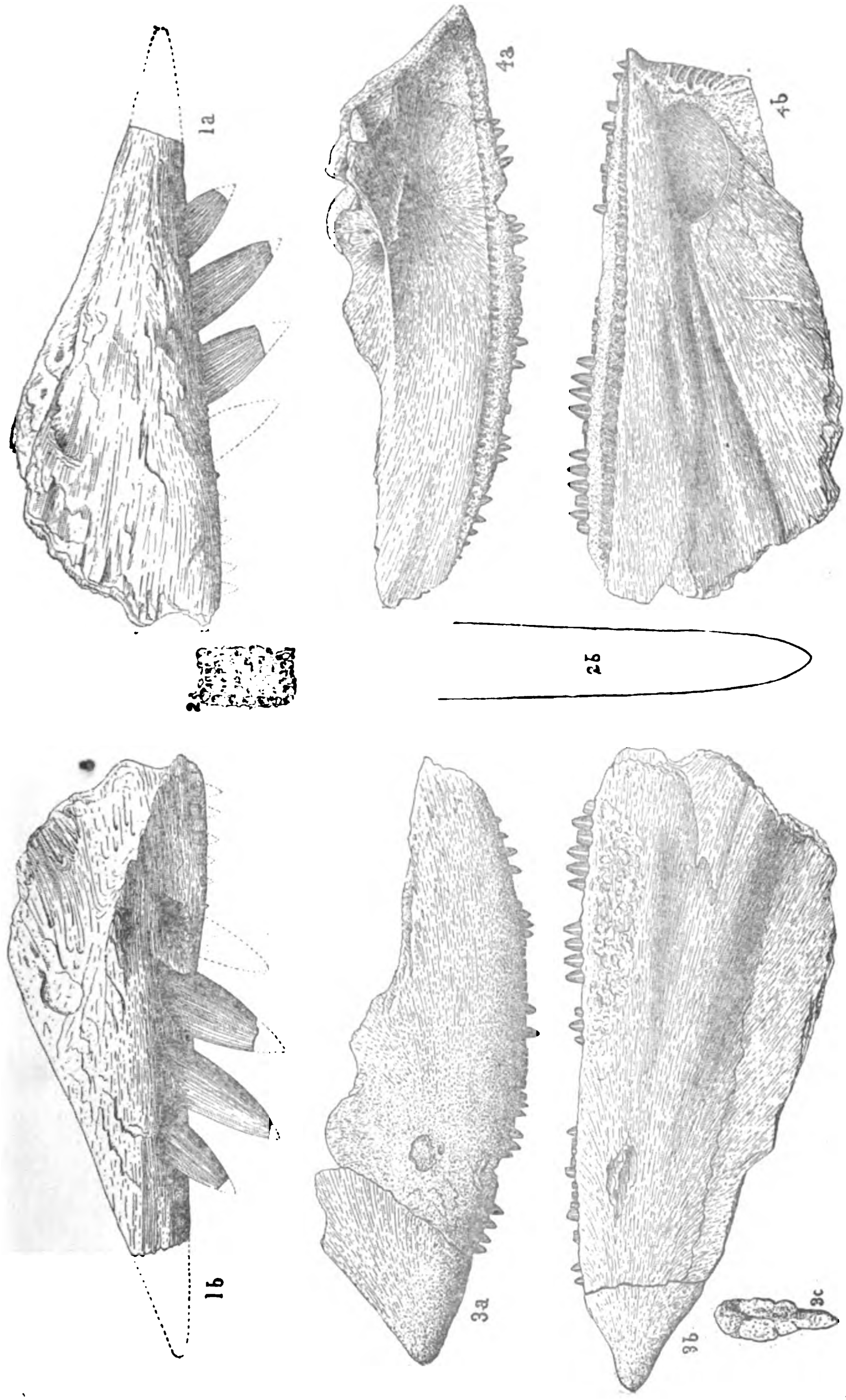
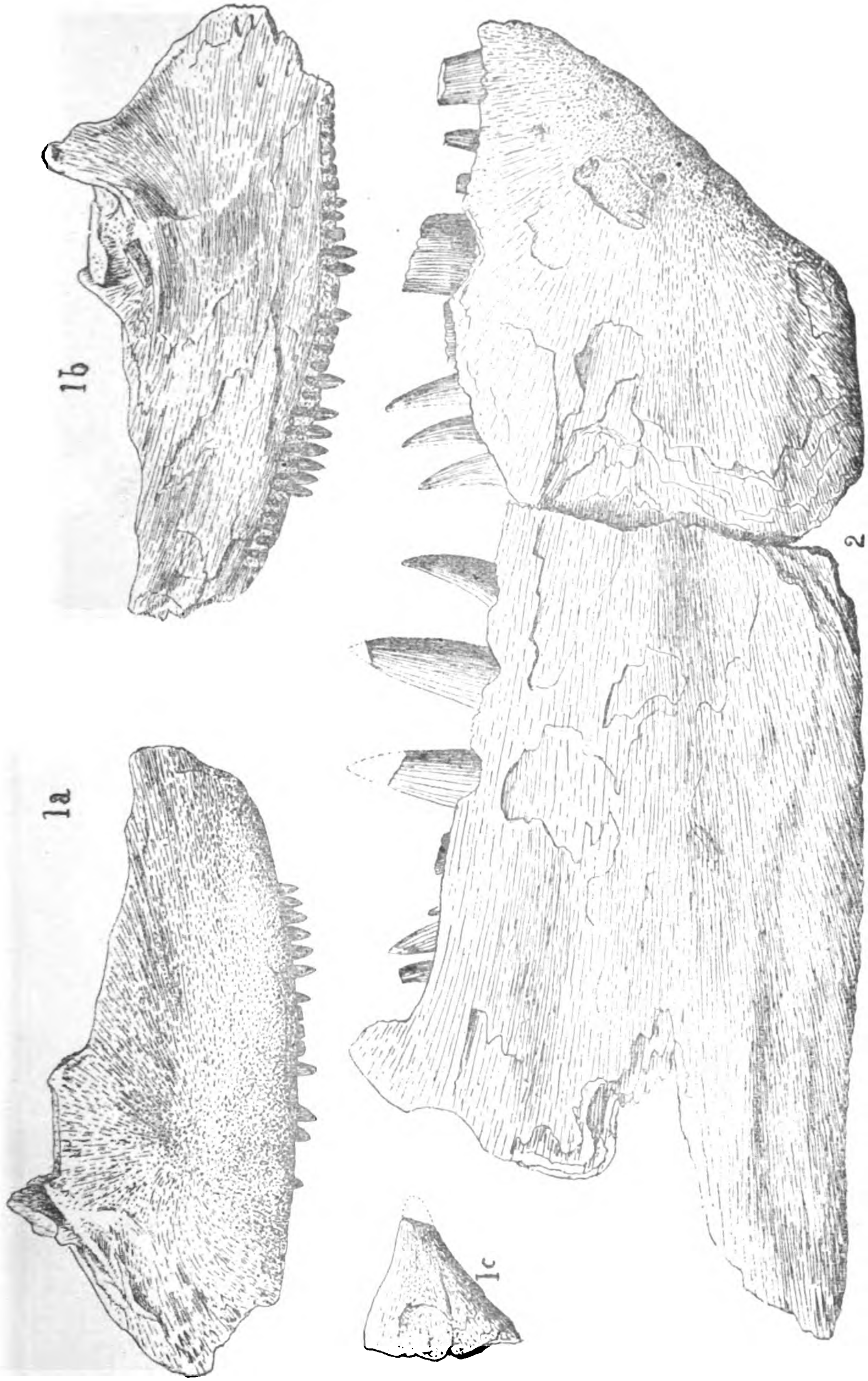

1a
4a
4b
2b
1b
3a
3b
3c

PLATE II.

Plate II. Figs. 1a, b, and c, External and internal views of maxillary, and external view of predentary of *Daptinus broadheadi* Stewart. Two-thirds natural size.

Fig. 2, Dentary of *Portheus lowii* Stewart. Two-thirds natural size.


1b
1a
1c
2

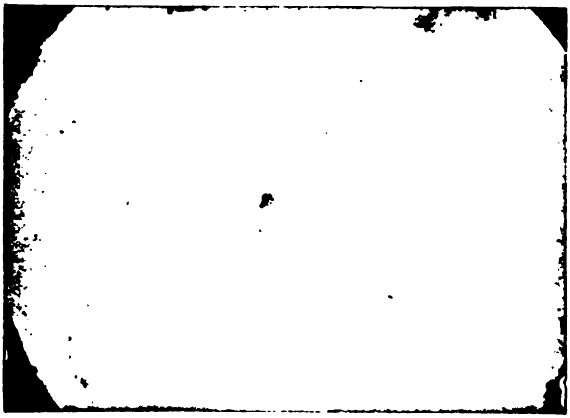


Fig. 1.

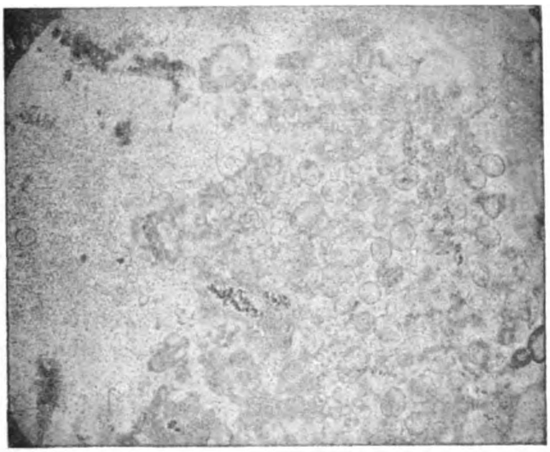


Fig. 2.

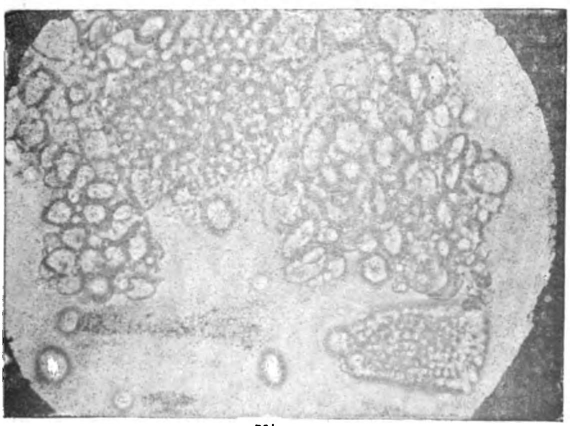


Fig. 3.

ADULTERATIONS OF BUCKWHEAT FLOUR.

Photomicrographs by C. E. McClung and M. A. Barber.

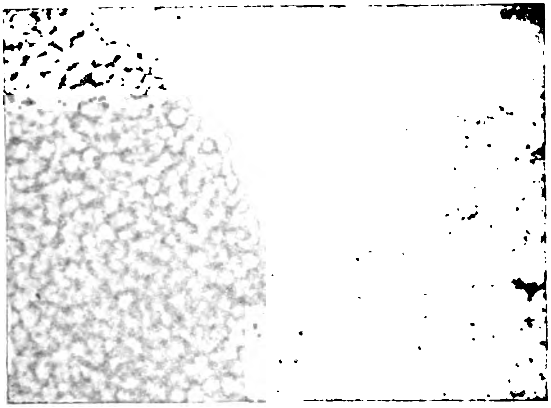


Fig. 4.

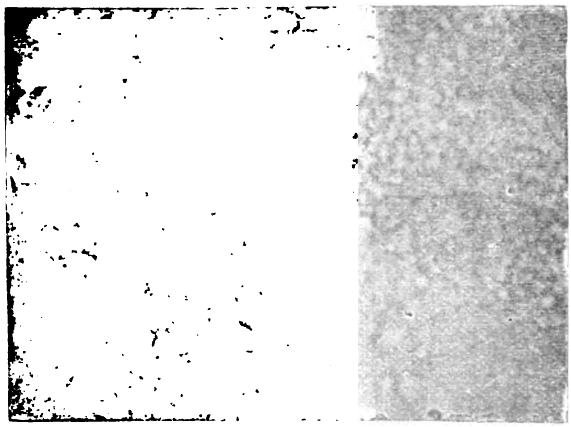


Fig. 5.

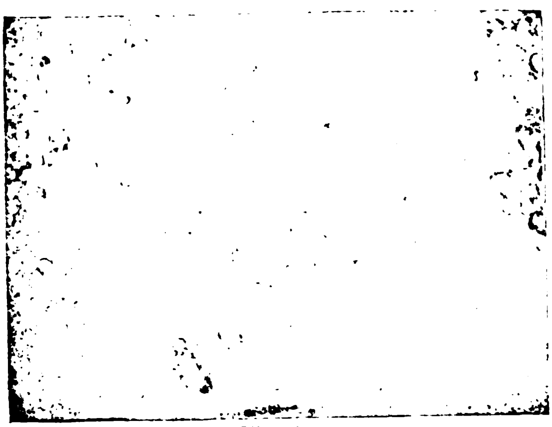


Fig. 6.

ADULTERATIONS OF BUCKWHEAT FLOUR.

Photomicrographs by C. E. McClung and M. A. Barber.

PLATE V.

The figures, which represent various stages in the division of the pollen mother cells of Asclepias cornuti, were drawn with a magnification of 1750 diameters by means of a camera lucida, and reduced to one-half of this magnification on the zinc plate.

Fig. 1, Young pollen mother cell.

Fig. 2, From mature pollen mother cell.

Fig. 3, Prophase of nuclear division of pollen mother cell.

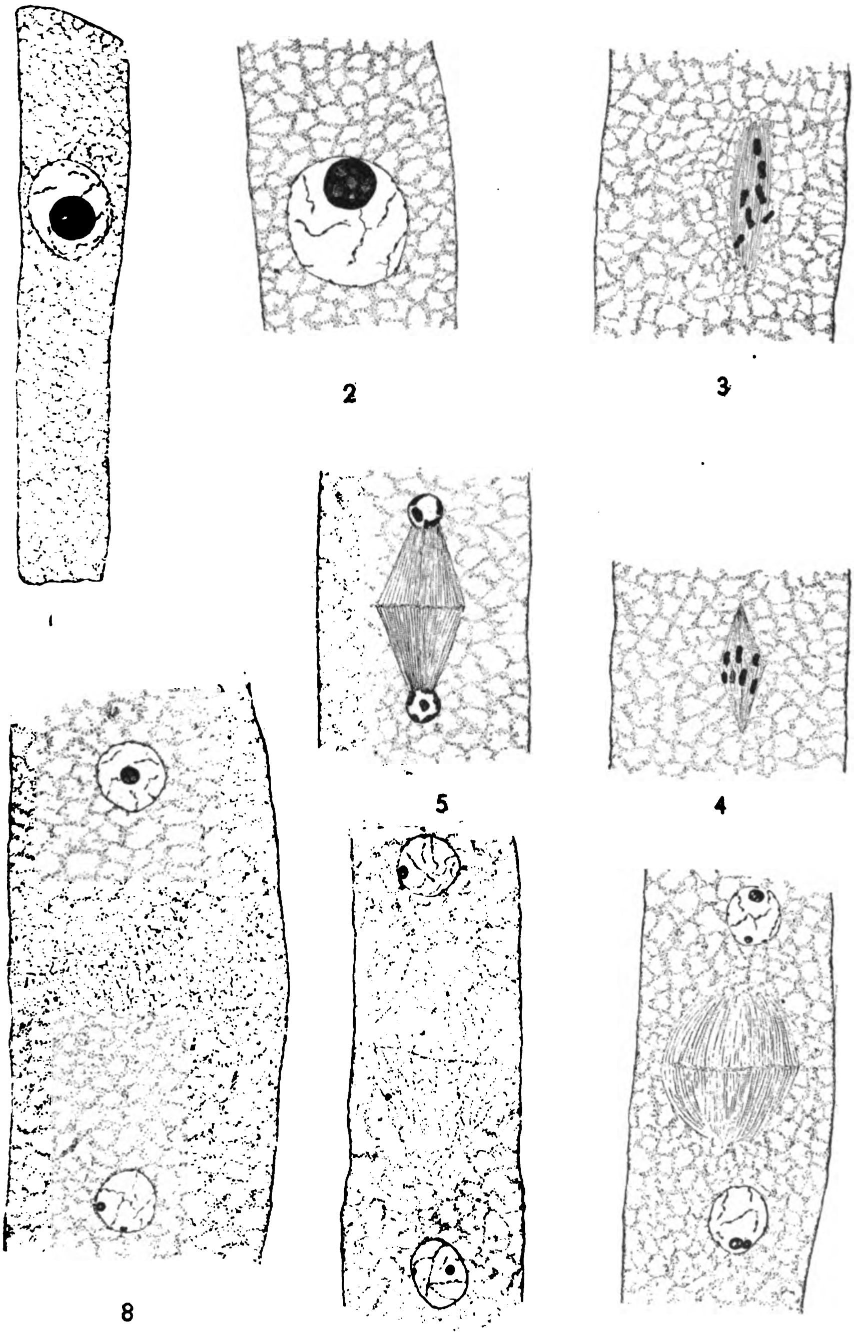
Fig. 4, Early metaphase of nuclear division of pollen mother cell.

Fig. 5, Anaphase of nuclear division of pollen mother cell. Cell plate in process of formation.

Fig. 6, Advanced stage in formation of cell plate.

Fig. 7, Completion of cell plate. Kinoplasm losing its thread-like structure.

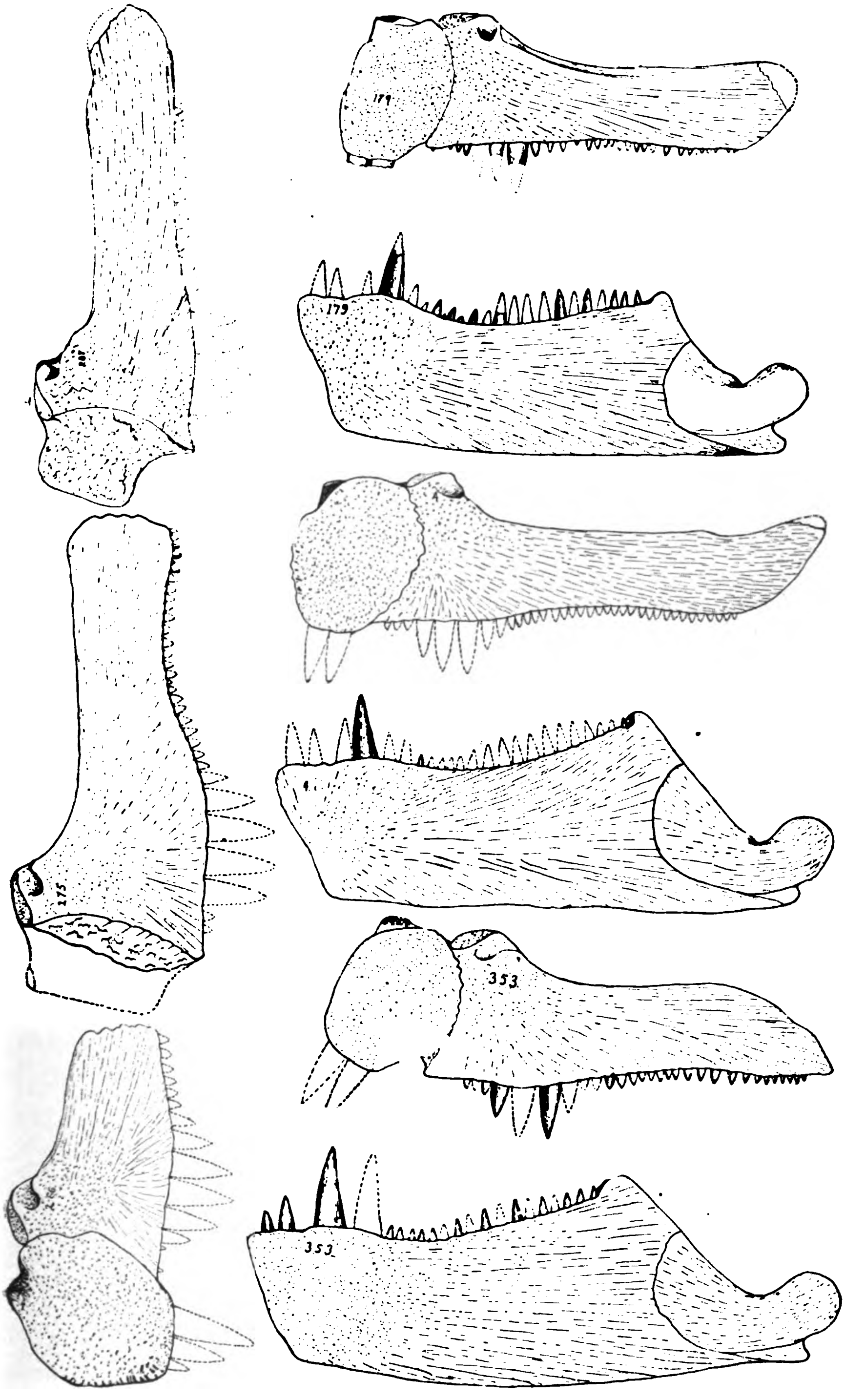
Fig. 8, Later stage in the disappearance of the kinoplasm.



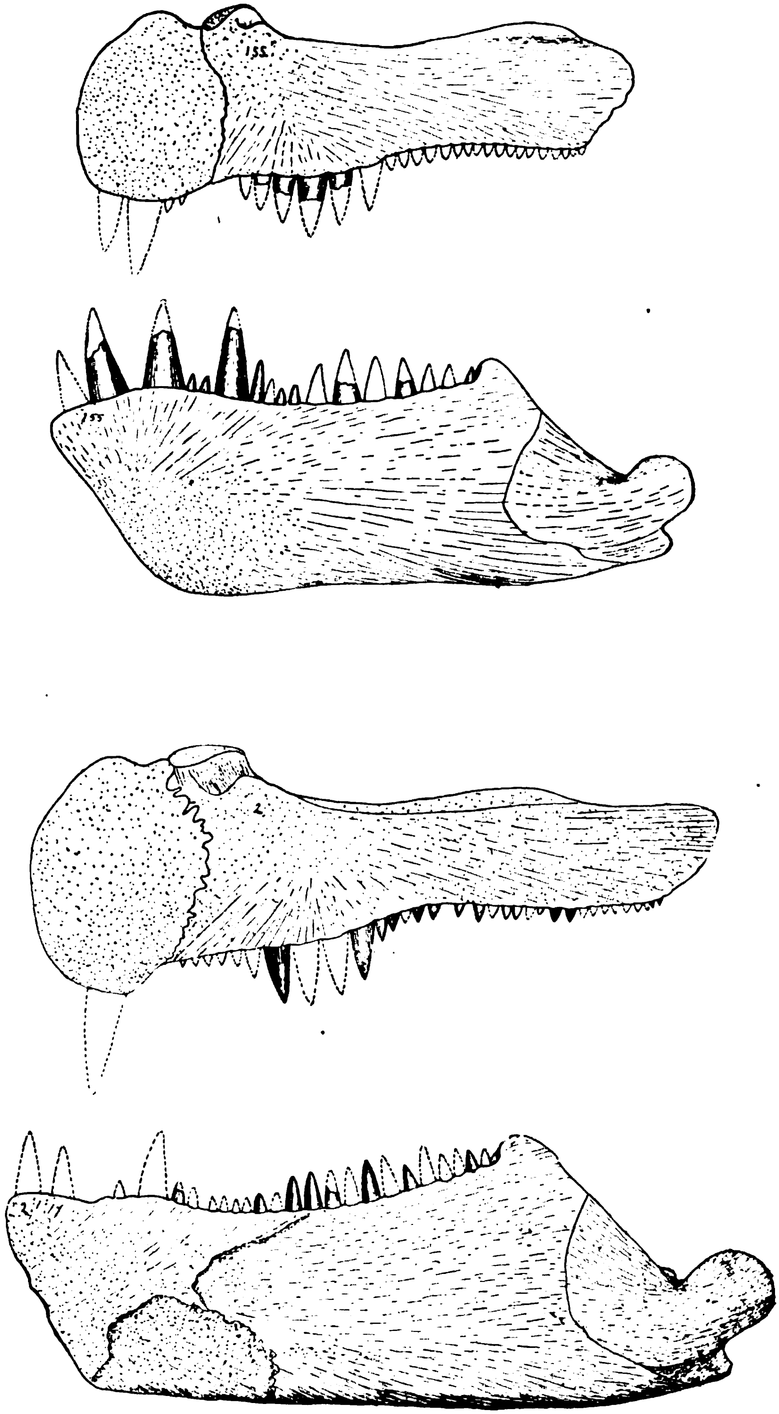
W.C.S. del.

PLATE VI.

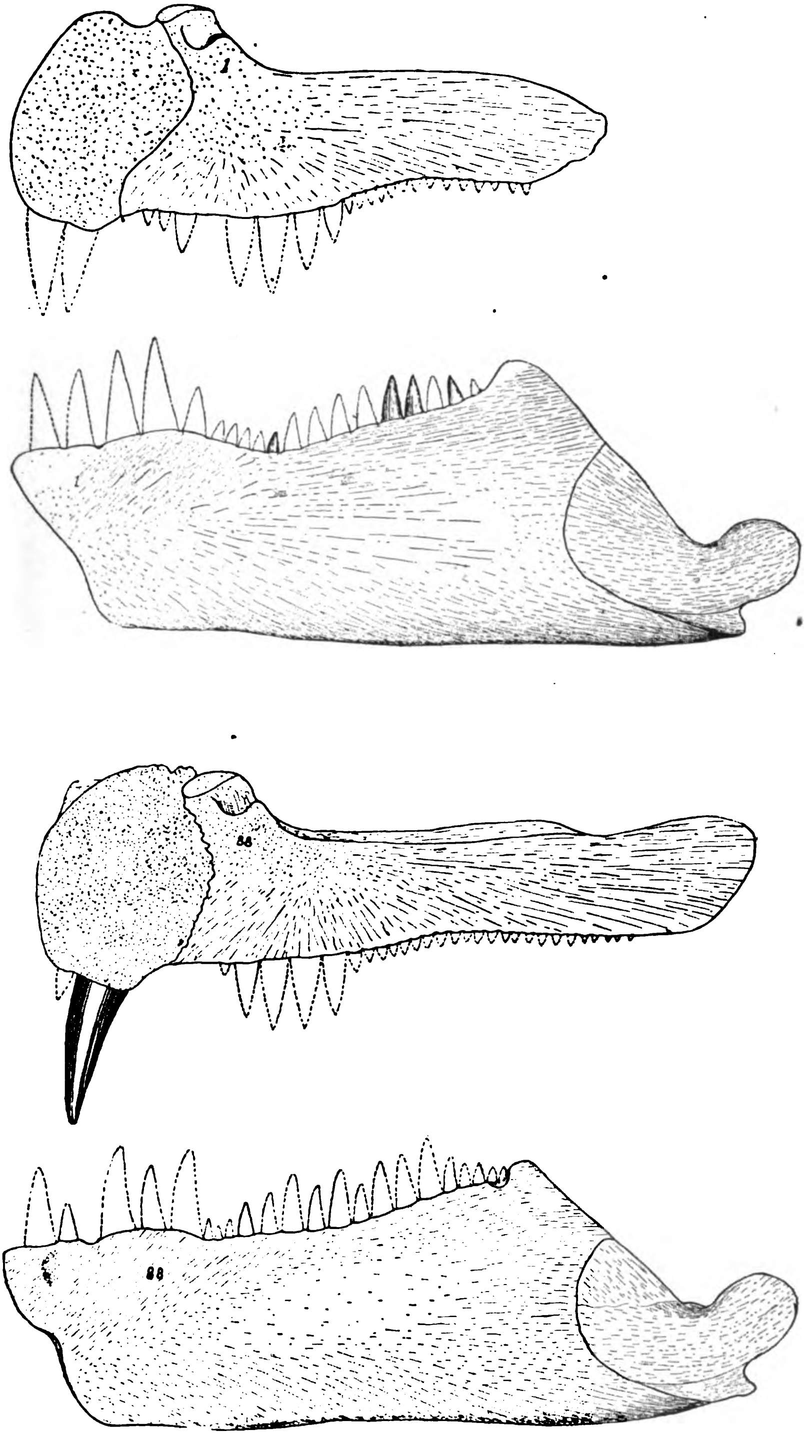
Figs. 1, 2, 8, 22, Attenuate forms.
Figs. 3, 4, 5, 6, 7, 23, Ventricose forms.
Fig. 9, Spires from which the jugum has been broken.
Figs. 10, 11, Two casts of the interior of the ventral valve.
Figs. 1, 3, 4, 21, 22, Show a marked fasciculation of striæ.
Figs. 19, 20, Exfoliated specimens.
Figs. 4, 8, 12, More general forms.
Figs. 15-31, Young specimens.



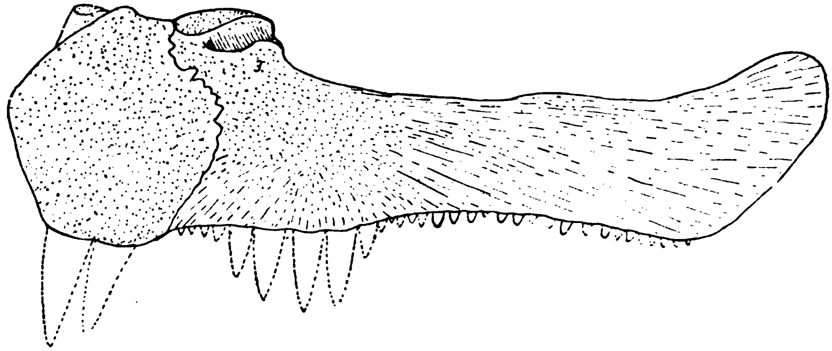
A. Stewart, from nature.

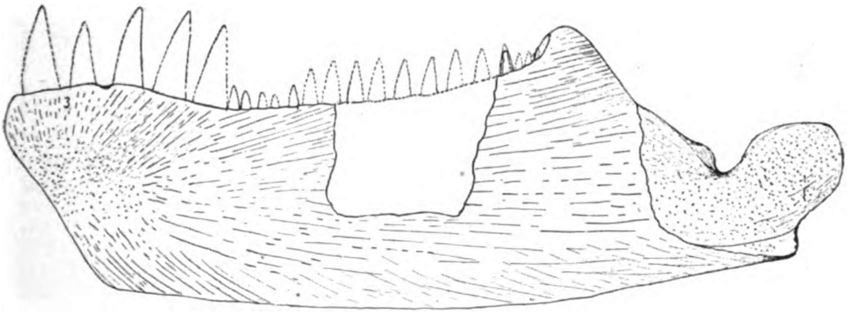


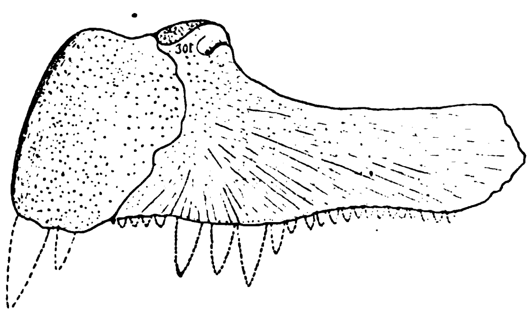
A. Stewart, from nature.

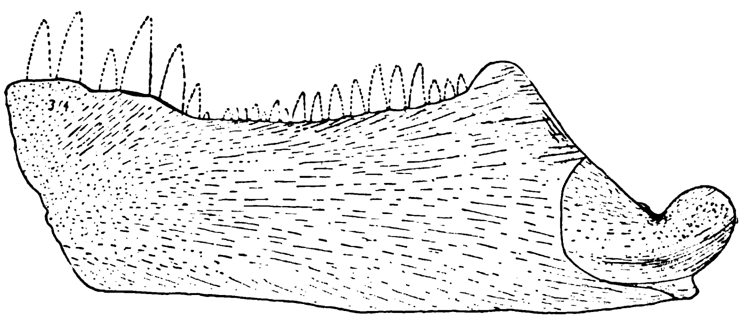


A. Stewart, from nature.









A. Stewart, from nature

PLATE XI.

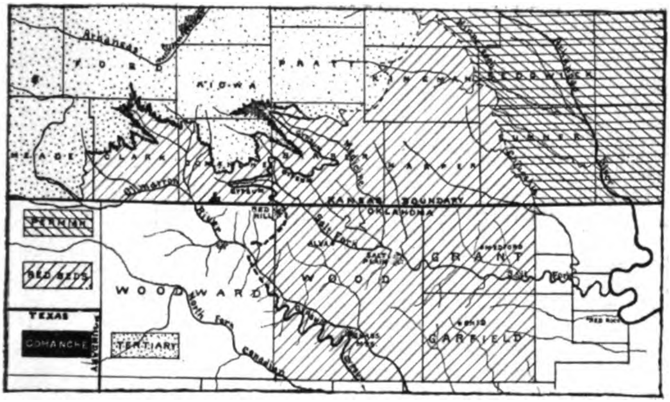
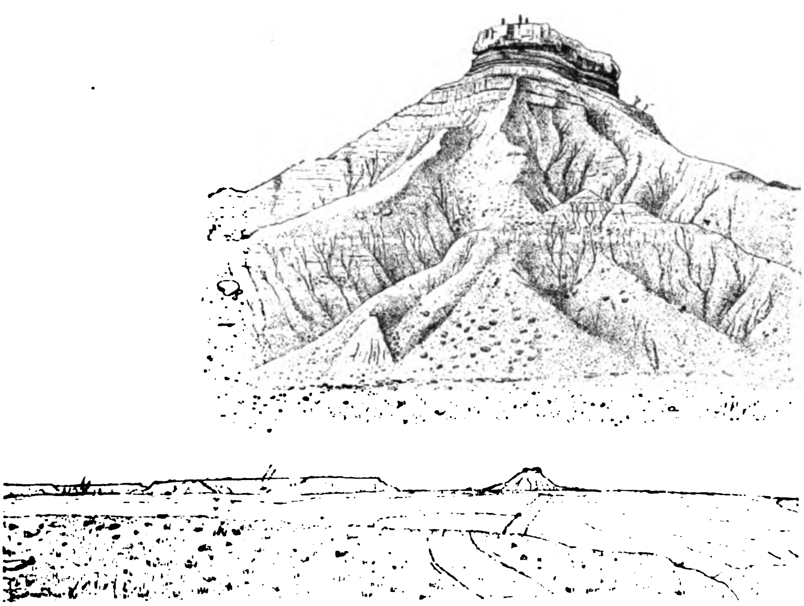


PLATE XII.



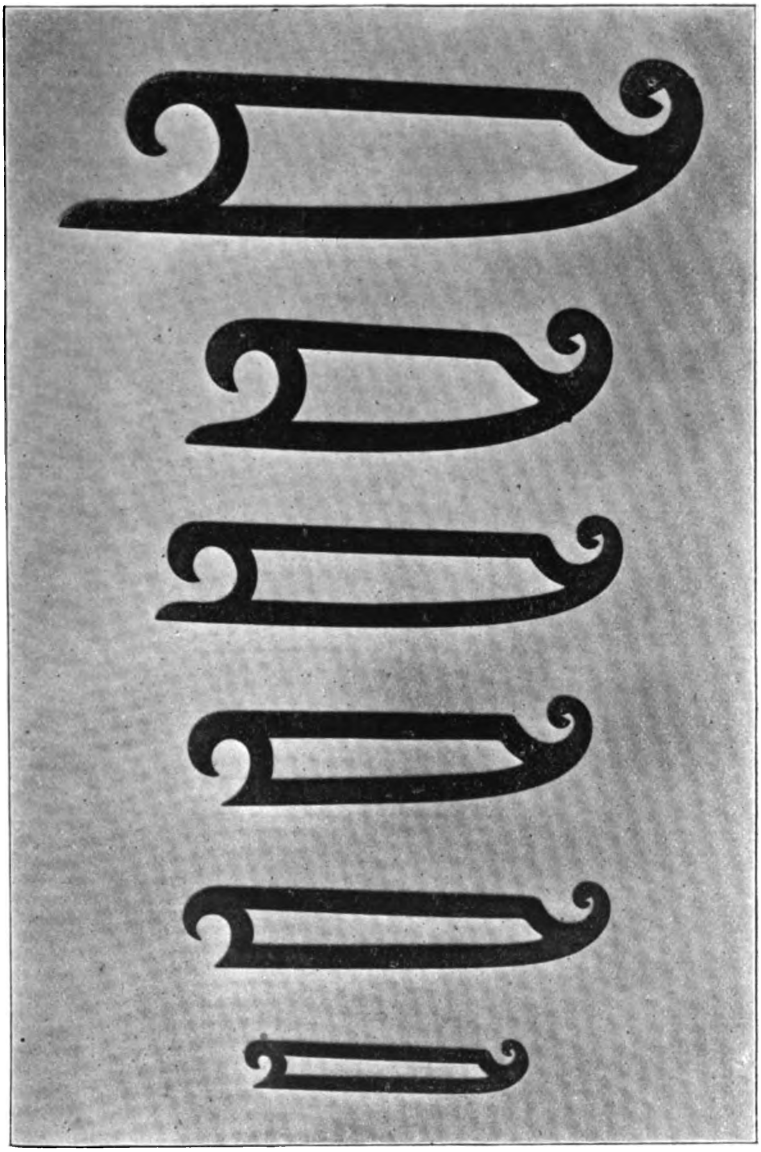
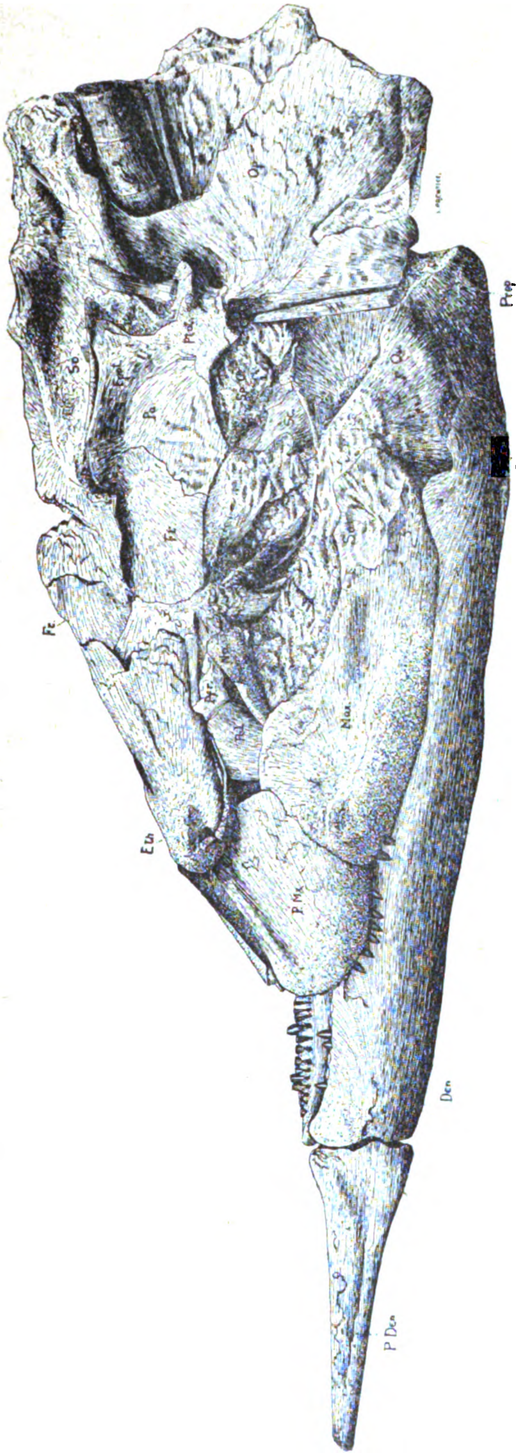


PLATE XIV.

Plate XIV. Skull of *Saurodon xiphirostris*, about three-fifths natural size.

PDen Predentary. *Den* Dentary. *DAr* Dermarticular of mandible. *Max* Maxilla. *PMax* Premaxilla. *Eth* Ethmoid. *Fr* Frontal. *PFr* Prefrontal. *Pa* Parietal. *SO* Supraoccipital. *EpOt* Epiotic process of parietal? *PtOt* Pterotic. *SpOt* Sphenotic. *HM* Hyomandibular. *Qu* Quadrate. *Na* Nasal? *PrOp* Praeoperculum. *Op* Operculum. *Pal* Palatine. *Sc* A portion of the sclerotic ring. *1, 2, 3.* Vertebræ.



S. Prentice, from nature.

PLATE XV.

Saurodon ferox.

Plate XV. Fig. 1. Upper and lower jaws, one-half natural size. *Max* Maxilla. *PMax* Premaxilla. *Den* Dentary. *PDen* Predentary. *DAr* Dermarticular.

Fig. 2. A small toothed element, the exact location of which is not known, natural size.

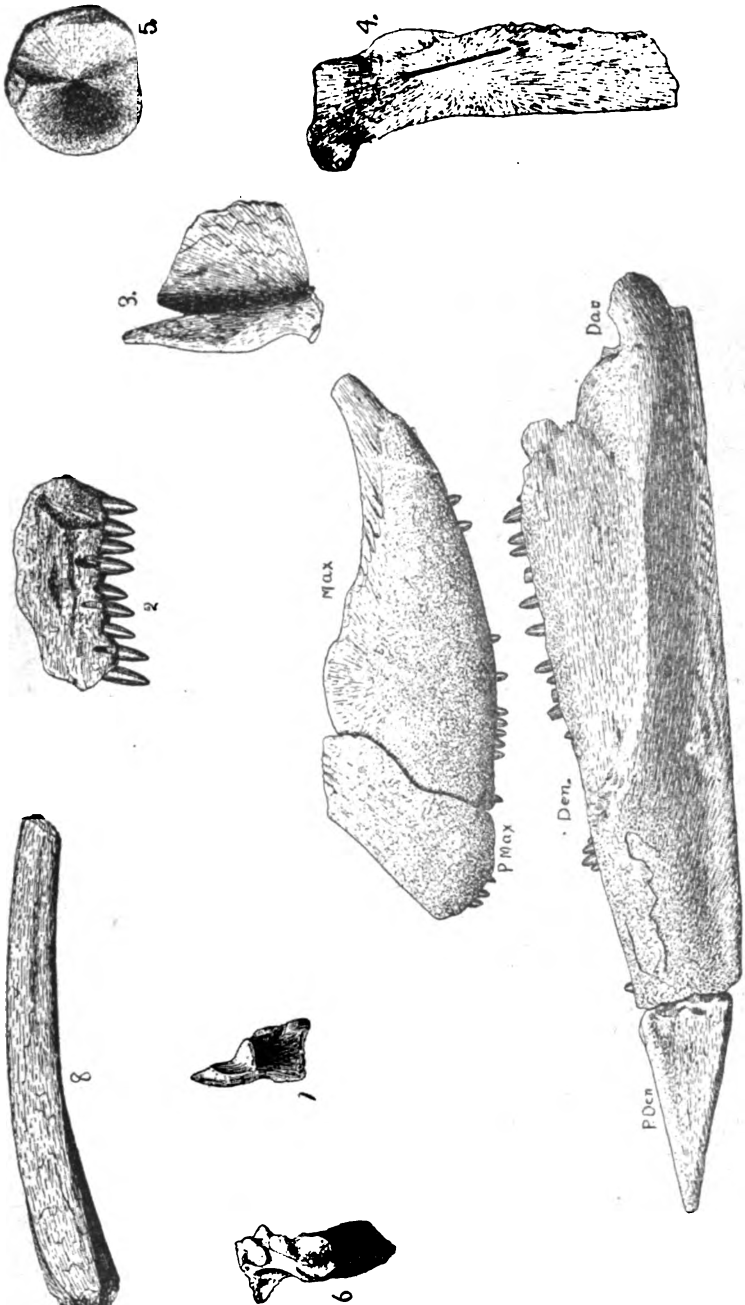
Fig. 3. Right quadrate, one-half natural size.

Fig. 4. Interoperculum?, one-half natural size.

Fig. 5. Centra of first anterior vertebræ from the front, natural size.

Fig. 6. Glenoid portion of scapula, natural size.

Figs. 7 and 8. Portions of fin rays.



S. Prentice, from nature.

PLATE XVI.

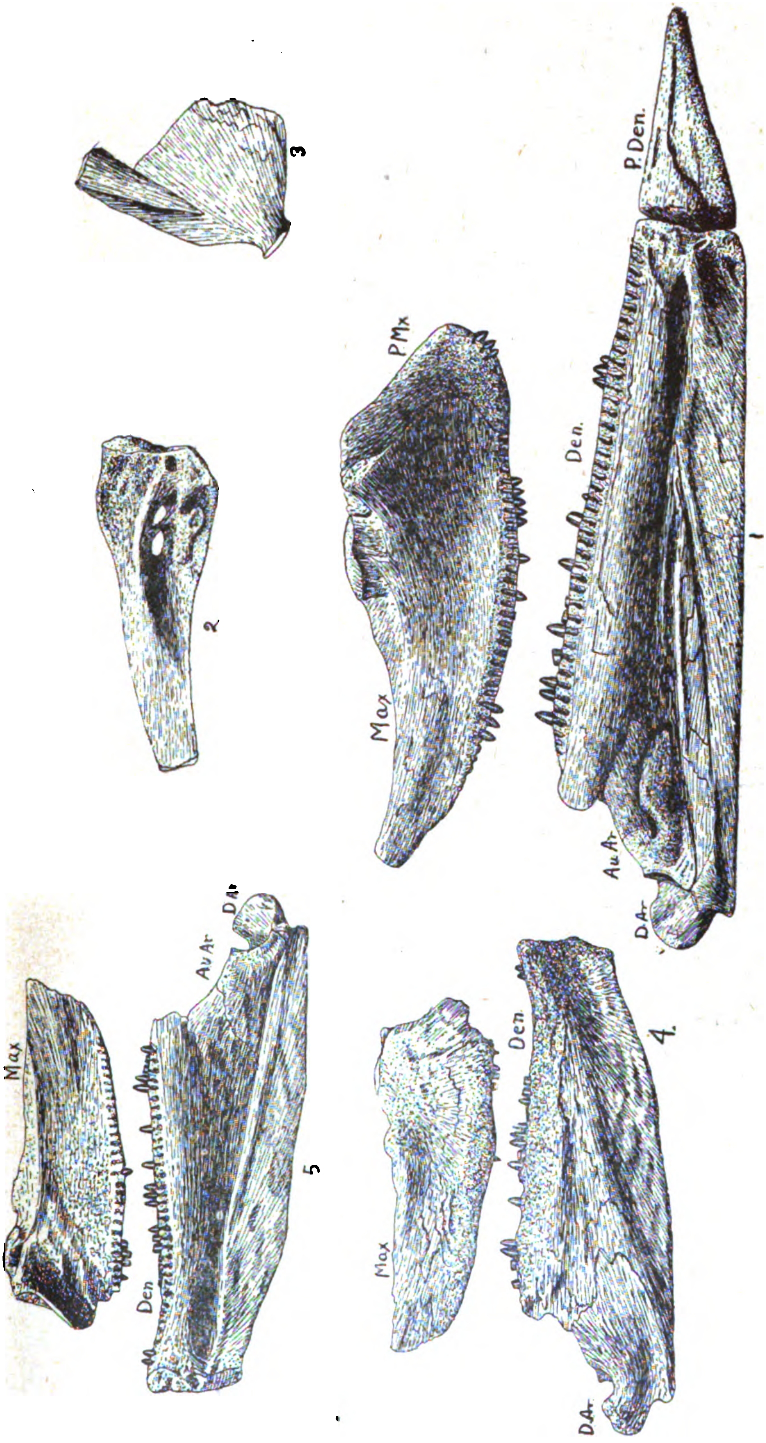
Saurodon farox.

Piate XVI. Fig. 1. Upper and lower jaws, one-half natural size. *Max* Maxilla. *PMax* Premaxilla. *Den* Dentary. *PDen* Predentary. *DAr* Dermarticular. *AuAr* Auarticular.

Figs. 2 and 3. Right quadrate and hyomandibuiar, internal side, one-half natural size.

Saurodon Phlebotomus Cope.

Figs. 4 and 5. External and internal view of the right maxilla and mandible, one-half natural size.

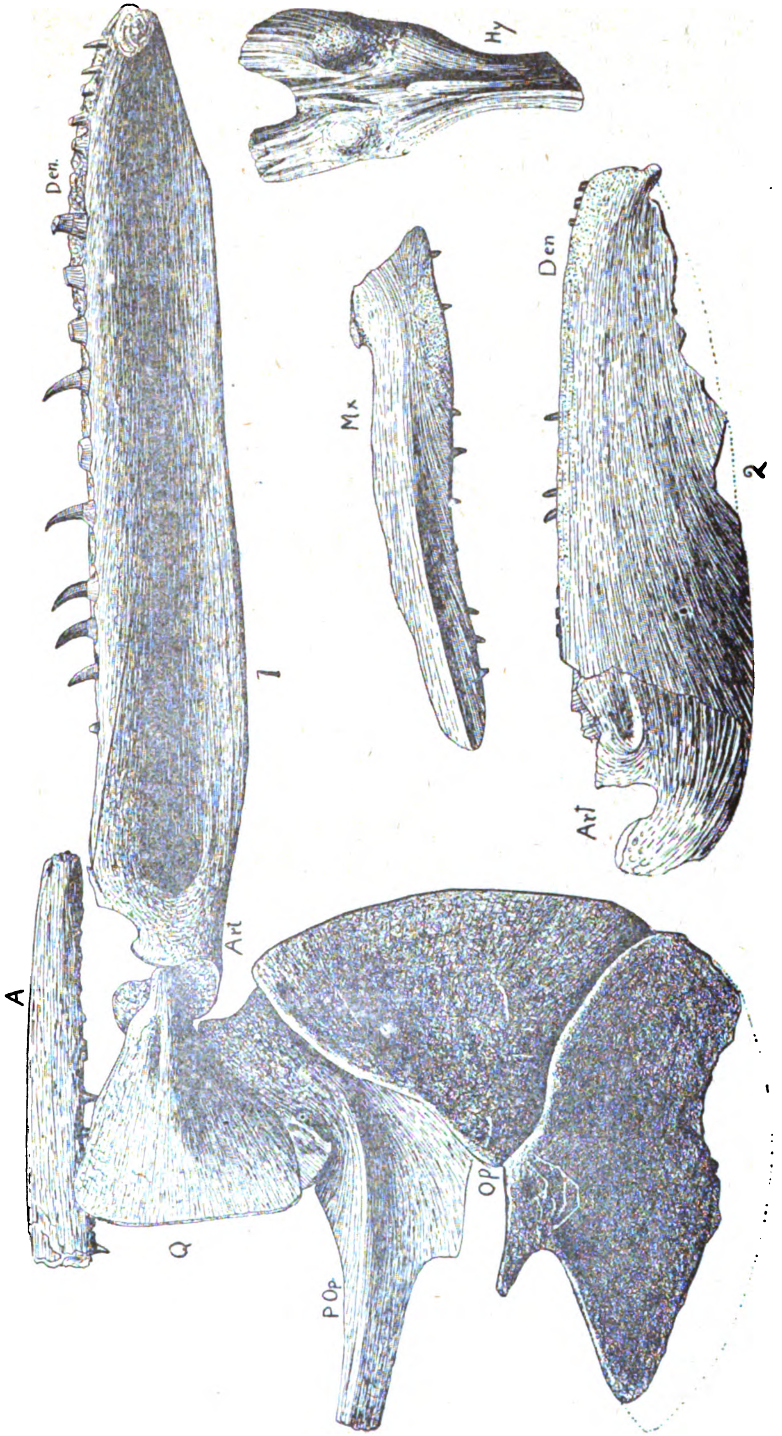


S. Prentice, from nature.

PLATE XVII.

Plate XVII. Fig. 1, *Pachyrhizodus leptognathus*, sp. nov. natural size. *Den* Dentary. *Art* Articular. *Q* Quadrate. *POp* Preoperculum. *Op* Operculum. *A* A small toothed element, the exact significance of which is not known.

Fig. 2. *Pachyrhizodus velox*, sp. nov. *Mx* Maxilla. *Den* Dentary, one-half natural size. *Hy* Supposed hyoid, natural size.



S. Prentice, from nature.